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AQUARIUS: A Modeling System for River Basin Water Allocation

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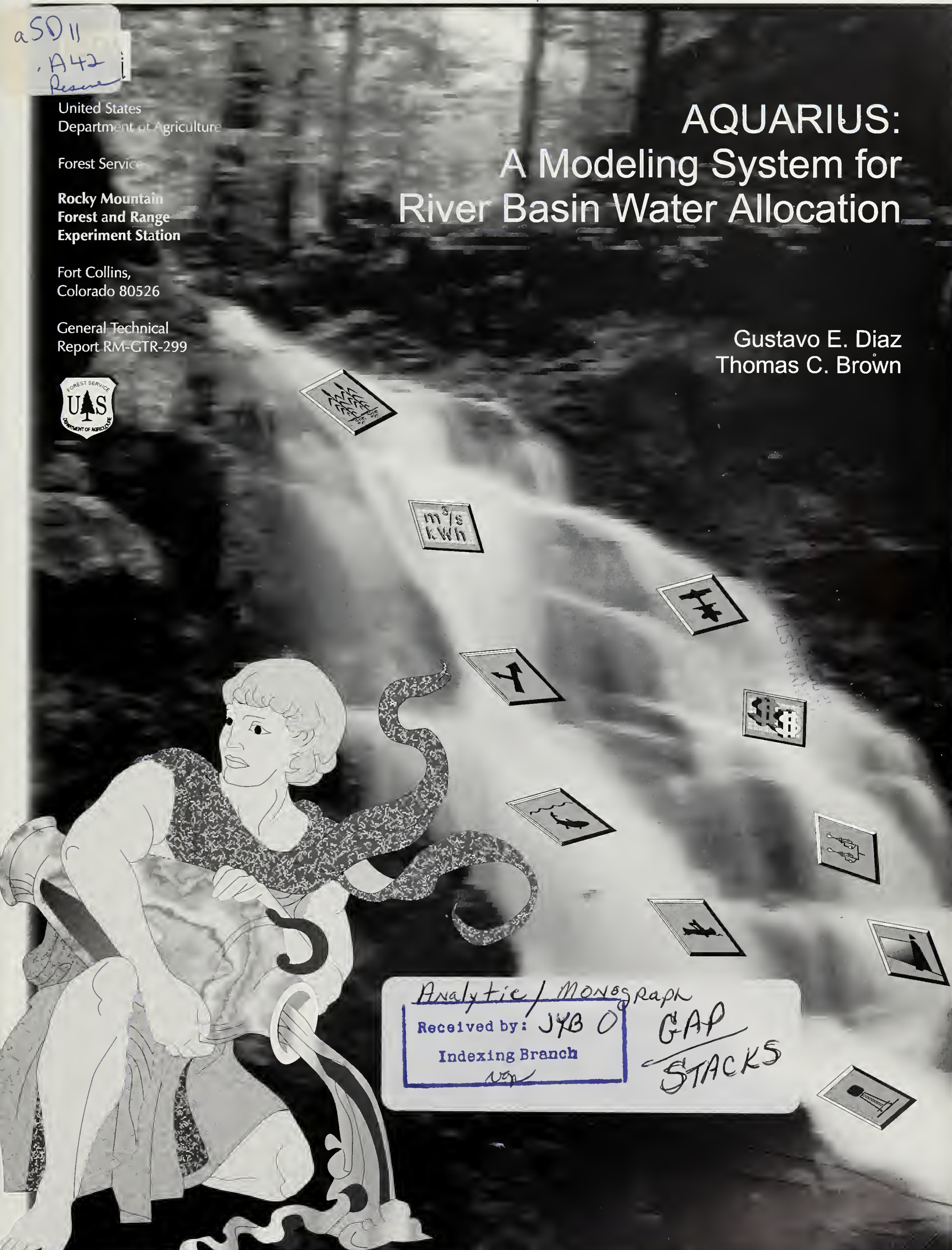

Abstract

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This report introduces AQUARIUS, a state-of-the-art computer model devoted to the temporal and spatial allocation of water among competing uses in a river basin. The model is driven by an economic efficiency operational criterion requiring the reallocation of stream flows until the net marginal return in all water uses is equal. This occurs by systematically examining, using a nonlinear optimization technique, the feasibility of reallocating unused or marginally valuable water storage and releases in favor of alternative uses. Because water-system components can be interpreted as objects of a flow network, the model considers each component as an equivalent node or structure in the programming environment as well. This is done using an object-oriented programing language (C++). The report contains a comprehensive description of the development of AQUARIUS, including how to use it and examples of its use.

Keywords: water allocation, economic efficiency, object-oriented programming, river-basin planning, economic demand, nonlinear optimization

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Contents

Executive Summary

This report describes the initial version (V96) of AQUARIUS, a state-of-the-art computer model devoted to the temporal and spatial allocation of water flows among competing traditional and nontraditional water uses in a river basin. The software runs on a personal computer under a Microsoft Windows 95 or Windows NT operating system. Usage is free for government agencies and for teaching and research purposes. Consult one of the authors regarding private use.

AQUARIUS is an analysis framework rather than a single dedicated model for water allocation. Future versions of the model are planned. The model was implemented using an object-oriented programming (OOP) language (C++). Water systems are ideal candidates for modeling under an OOP framework, where each system component (e.g., reservoir, demand area, diversion point, river reach) is an object in the programming environment.

In V96, an economic efficiency criterion was adopted for determining water allocation because economic demands play a key role in water allocation decisions, and because of the greater accessibility of economic value estimates for nontraditional water uses such as recreation. This decision criterion calls for reallocating stream flows until the marginal returns in all water uses are equal. Each traditional use and nontraditional use is, if possible, represented by a demand curve (i.e., a marginal benefit function) that is characterized by an exponential function.

For a water use with a predetermined level of allocation but without a defined economic demand function, the analyst can either constrain the model to meet the specified allocation or experiment with surrogate demand curves until the required level of water allocation is reached. The latter approach indicates the level of economic subsidy required to provide the incremental increases of flow to sustain the use in open competition with other uses. The interactive nature of AQUARIUS facilitates such experimentation.

The water allocation problem solved by AQUARIUS, involving a set of exponential demand functions, requires a complex nonlinear objective function. The solution technique in V96 uses the special case of the general nonlinear programming problem that occurs when the objective function is reduced to a quadratic form and all the constraints are linear. The method approximates the original nonlinear objective function by a quadratic form using Taylor Series expansion and solves the problem using quadratic programming. A succession of these approximations is performed using sequential quadratic programming until the solution of the quadratic problem reaches the optimal solution.

The user interacts with the model through the network worksheet screen (NWS), which represents the water system of interest using the inherent capability of the object-oriented paradigm for graphical representation. In the NWS, each water system component corresponds to an object—a graphical node or link—of the flow network. These components are represented by icons, which are pictorial representations of the objects. By dragging and dropping these icons from the menu, the model creates instances of the objects on the screen. Components can be

repositioned anywhere in the NWS or can be removed. Once nodes (e.g., reservoirs, demand areas) are placed, they are linked by river reaches and conveyance structures. This operation occurs by left-clicking on the outgoing terminal of a node and then on the incoming terminal of the receiving node. This procedure facilitates the assembly or alteration of water systems by connecting their system components in the NWS. The creation and alteration of flow networks is further facilitated by copying and inserting an object or whole portions of an existing network onto the same or a new NWS. Copy/paste creates new instances of the object(s) and duplicates their data structure, creating clones of the original objects.

The model's input data have been divided into physical and economic data. The physical data include the information associated with the dimensions and operational characteristics of the system components, such as maximum reservoir capacity, percent of return flow from an offstream demand area, and powerplant efficiency. The economic data consist mainly of the demand functions of the various water uses competing for water.

Although V96 implements only a monthly time step, AQUARIUS was conceived to simulate the allocation of water using any time interval, including days, weeks, months, and time intervals of nonuniform lengths. Future versions of the model will support these other time steps.

AQUARIUS can be used in a full deterministic optimization mode, for general planning purposes, or in a quasi-simulation mode, with restricted foresight capabilities. The model distinguishes between the period of analysis, used to specify the length of the whole segment of time for which the model will simulate system operation, and the optimization horizon, used to specify how far into the future the model should look to build the optimal operational policies. Setting the optimization horizon equal to the period of analysis produces a full-period optimization.

Formulating a water allocation problem entirely within the domain of the objective function allows the user to redirect the water allocation process in any direction in real time, directly from the screen, as the optimization progresses. This unique feature provides an expeditious and innovative mode of exploring what if... scenarios.

Under the dominant water allocation doctrine in the Western United States—the prior appropriation doctrine—the water available for a new application is reduced by the sum of all prior established rights. A priority-based allocation in a heavily appropriated river can become inefficient as values change if institutional barriers or market failures impede voluntary transfers of rights from lower-valued to higher-valued uses. Because institutional barriers and market failures commonly affect Western water, actual water allocations may be different from the economically efficient allocations achieved using V96.

It may be helpful to compare the actual allocation with an efficient allocation. Such a comparison may indicate promising opportunities for private water trades or, where such trades are hampered or precluded by institutional barriers, may indicate areas where institutional reforms can allow for a more efficient water allocation. Where water developments are publicly financed, the comparison may indicate directions that the public entity should consider to increase the efficiency of the project. V96 facilitates such comparisons by characterizing an efficient allocation, subject to the analyst's ability to specify demand functions for the key water uses.

A simulation capability is envisioned for the next version of the model. With this capability, the current allocation could be characterized within the same modeling structure as that used to articulate an efficient allocation. This will allow a more direct comparison of the simulated current allocation with the efficient allocation than does V96.

A copy of this document can be obtained by contacting the second author at the Rocky Mountain Research Station in Fort Collins. The latest distribution of the program AQUARIUS, together with a short version of this technical document, can be downloaded from the World Wide Web at: http://www.engr.colostate.edu/depts/ce and going to the software section.

Chapter 1

Introduction

Water resource and environmental managers face increasing pressure to recognize traditional as well as nontraditional water uses. Most water management systems were designed and are typically operated for traditional water uses, including flood control, hydropower, irrigation, and urban water supply. Nontraditional uses include preserving of the geomorphological and biological integrity of a river, as well as providing opportunities for water-based recreational activities. To effectively integrate nontraditional water uses into a water management system, improved analytical tools are needed. This report describes such a tool. AQUARIUS is a river basin model that integrates traditional and nontraditional water uses using the most advanced computer programming language and graphics capabilities. This report is a comprehensive description of AQUARIUS including how to use it and examples of its use.

Modeling River Basin Systems

Concern for the environment, demand for outdoor recreation, and interest in sustainable development are redefining how water is stored and distributed in river basins. In particular, tradeoffs between instream and offstream water uses have become increasingly important in planning and managing water resources. These tradeoffs are important in new water developments as virtually all water projects have an impact on recreation and environmental quality. However, they are also important for existing water developments, especially when they are re-evaluated for license renewal. Such concerns require modeling to determine how water used for traditional activities and that used for nontraditional activities affect each other.

Similar to the systems they were designed to analyze, river-basin models have focused on traditional water uses. Even multipurpose operation models usually included only traditional purposes such as hydropower and diversions to farms or cities. Nontraditional water uses, to the extent they were incorporated in models, were considered of secondary importance. The infancy of the environmental sciences has contributed to the diminished significance of nontraditional water uses in modeling efforts.

The degree that nontraditional water uses have been incorporated into river-basin models has been limited by the perceived lower importance of nontraditional uses, lack of knowledge about geomorphology and riparian ecosystems, and by the difficulty of measuring the benefits of nontraditional uses. The values of most traditional uses are quantifiable in terms of a benefit function that relates resource availability to the benefits generated. However, benefits of nontraditional uses are generally not estimable in units commensurate with the traditional uses, so they are often omitted from quantitative reservoir analyses and operations.

Over the past 30 years, the increasing value of outdoor recreation and other amenities has encouraged economists to develop techniques to estimate the economic value of nonmarket goods and services. These methods have been applied to water uses including activities that rely on instream flow. This has created the opportunity for enhanced models of water systems that provide continued support of the traditional demands and also recognize nontraditional water uses for environmental, recreational, and aesthetic objectives.

Also over the last three decades, systems engineering and, in particular, mathematical modeling have increasingly been used for design, planning and operation of complex water resource systems. During the planning stage, mathematical models allow decision makers to evaluate the physical and economic impacts of existing and alternative structural measures, changes in allocation policies, increased demand levels, and new environmental and institutional restrictions. Used in the day-to-day operation of a water system, mathematical models provide guidance to operators who make real-time decisions concerning the quantity of water to be stored in or released from reservoirs, the amount of reservoir withdrawals, and offstream diversions. These decisions are all based on known or forecasted inflows and water demands in the basin.

Two basic categories of water resource models are simulation and optimization models. A simulation model is a conceptualization of a water system used to predict its hydrologic response and, in some cases, its economic performance under predefined operational conditions. Such models are based on water-balance accounting procedures for tracking the movement of water through a network of system components. Separate executions of a simulation model are generally needed to compare the response of a system under alternative system configurations, operating rules, demand levels, or natural flow sequences. The outputs from these multiple runs are evaluated by comparing the resulting time series of storage levels, reservoir releases, hydroelectric production, water supply shortages, water quality parameter values, etc. Some elements of optimization may be embedded within a reservoir system simulation model to perform certain specific tasks. Simulation models have typically been created from scratch to respond to the peculiarities of a given system, which precludes their reusability to model another water system. However, some general simulation models designed to be adapted to the conditions of a specific river basin are available, notably HEC-5, which was developed by the U.S. Army Corp of Engineers (1982).

Whereas simulation models are limited to predicting system performance for a user- specified set of decision rules or priorities, optimization models automatically search for an optimal solution to the water allocation problem. Optimization refers to a mathematical formulation in which a formal algorithm solves for values of a set of control variables that minimize or maximize an objective function, subject to pre-established operational restrictions. The objective function and constraints are represented by mathematical expressions as a function of the control variables.

In hydrologic optimization models, water inflows and demands are typically modeled deterministically; that is, they are known for all time periods of interest. The ability of an optimization model to base a decision (e.g., to allocate water) at any given time interval on

foreknowledge of future inflows and water demands makes an optimization model distinct from a simulation model. However, an optimization model can be implemented one time period at a time, with the result used to determine the initial conditions for optimization of the next time period. When used this way, the optimization model performs somewhat like a simulation model.

Different types of objective functions may be used to measure system performance including utility and penalty functions or, preferably, mathematical expressions of a planning or operational objective. Several single objectives can be combined into a global objective function for the water system if all objectives can be expressed in commensurable units. When this is not possible, one of two approaches is usúally adopted. The first approach is to use the primary objective as the objective function while treating the secondary objectives as constraints at fixed user-specified levels. The second approach is to construct a global objective function as a weighted sum of the different single objectives. Note that all optimization models have an embedded system simulation model on which the optimization acts. Because of the mathematical complexity involved in formulating optimization problems, general optimization models, designed to be easily adapted to the conditions of a specific river basin, have until now, been unavailable.

Elements of simulation and optimization models for water systems analysis are found in a third category of models, called network-flow models. Simulation models based on network-flow programming are a hybrid formulation that combines some advantageous features of simulation with some optimizing capability. Their simulation capability is limited to situations where specified quantities of water are to be stored or allocated to different users according to certain priorities expressed as weighing factors of a linear objective function. Variable priorities for a water use, such as those represented by a downward sloping demand function, can only be characterized in such models by stepping through a series of discrete levels that approximate the function. An important limitation of typical applications of the network-flow approach is that the optimal allocation of water is determined for each time interval of analysis independently rather than in a fully dynamic sense, as is typically required by multi-period problems in water resources.

Rogers and Fiering (1986) maintain that the most valuable role of systems analysis in water allocation is generating viable new alternatives to solve the water allocation problem. This is most effectively attained by using fully-dynamic optimization models. The series of alternatives, whose characteristics might be distinct, may form the basis of negotiation toward allocation of stream flows until the net marginal return in all water uses is equal. The key concept is "generation of new alternatives" rather than using the computer to track sequences of hydrologic input through the system, which only enforces already existing operation rules. When the analysis is carried out under the suggested framework, the techniques of systems analysis have the potential of significantly improving water resources planning and management.

Objectives

Our primary objective was to develop a water allocation model driven by an economic efficiency criterion that called for reallocating stream flows among traditional and nontraditional uses, subject to specified constraints, until the net marginal economic returns in all water uses was equal. We investigated economic and systems engineering techniques by which multipurpose river basin systems could be analyzed to reach an optimal allocation of flows. We adopted an economic criterion for determining an optimum, primarily because economic demands have traditionally played a key role in water allocation decisions and because economic value estimates for some nontraditional water uses are now available.

The modeling approach we adopted identifies tradeoffs between water uses by systematically examining, using a nonlinear optimization technique, the feasibility of reallocating water to alternative uses. The values of water in most uses are represented by economic value functions that express society's measured or estimated willingness to pay for different water uses. The model solution will indicate society's marginal willingness to pay for water given the optimal water allocation, subject to the constraints imposed on the solution. The economic opportunity cost of allocating water to uses for which economic demand functions could not be estimated, such as the value of protecting an endangered fish species, would also be indicated.

For the purpose of planning and operation of multi-reservoir, multi-purpose water systems, we assessed the usefulness and adaptability of the following optimization techniques: Sequential Linear and Sequential Quadratic Programming, Dynamic Programming, Optimal Control Theory, and Extended Linear Quadratic Gaussian Control. We found that the optimization techniques most commonly used in reservoir-system optimization models are Linear Programming and Dynamic Programming, although nonlinear programming methods, including Optimal Control Theory, are also used. Although our modeling framework allows implementation of various solution algorithms, to date we have implemented only one algorithm category (sequential-approximation, a category within the realm of convex-programming) to solve nonlinear programming problems. Sequential-approximation algorithms, which include linear-approximation and quadratic-approximation methods, were selected because they can effectively handle the physical and economic nonlinearities found in real-world water resources allocation problems. Although sequential-approximation methods have been available for some time, because of their demand for computing time and memory, they have only recently become a practical option in personal computing.

Our secondary objective was to develop an easy to use model. A common problem of many river-basin models is that they are cumbersome to construct and amend. To improve this, we used the latest advances in programming languages to develop the software architecture and to provide graphical interfaces that simplify specifying the river basin components, entering the data, and interpreting the results.

AQUARIUS is a state-of-the-art computer model devoted to the temporal and spatial allocation of flows among competing water uses in a river basin. The model is named after the ancient Greek personification Aquārius, who, according to the Oxford University Press dictionary, was the "overseer of the public water supply" or the "superintendent for the supply of water." The first version of the software is version 96 (V96).

Among the large number of journal publications, books, and technical reports reviewed during this research, two classical references stand out: *Economics of Water Resources Planning* by James and Lee (1971), and *Water Resource Systems Planning and Analysis* by Loucks, Stedinger and Haith (1981). Both were important sources of inspiration.

Report Organization

The remainder of the report is organized as follows:

Chapter 2	presents the current water allocation systems in the United States and introduces the concept of water allocation under economic efficiency. This chapter also presents a complete overview of water demand functions for use in optimization modeling.
Chapter 3	introduces the operation and modeling assumptions adopted in the model for each of the water-using sectors possible in a basin.
Chapter 4	derives the economic benefit functions from all potential water users in a river basin and shows how are they quantified for the purpose of the model.
Chapter 5	presents the general approach to the solution of the water allocation problem as implemented in AQUARIUS V96. This chapter also provides the general mathematical formulation and details of the optimization technique.
Chapter 6	justifies the use of an object-oriented programming (OOP) framework for modeling water resources systems. This chapter also introduces the reader to some basic concepts of OOP and describes how they were used in the current modeling effort.
Chapter 7	familiarizes the reader with the use of the model by explaining the tools available for creating new flow networks, providing the necessary inputs, and finding and displaying optimal water allocations for the current problem.
Chapter 8	presents a hypothetical case study for a system of intermediate complexity. Besides describing the optimal system operation, it focuses on network subcomponents to further illustrate model capabilities.
Appendix A	introduces the mathematical derivations related to the adopted solution method.
Appendix B	presents the procedures for fitting exponential demand function based on user provided economic information.
Appendix C	presents an alternative approach for the explicit consideration of reservoir evaporation in the formulation of the model.

Chapter 2

Efficient Water Allocation

Much has been written about economic efficiency and water allocation (e.g., Hartman and Seastone 1970, Howe 1971, James and Lee 1971, Water Resources Council 1983, Young 1996, and Young and Gray 1972) and about economic efficiency and optimization (Baumol 1977). In this chapter, we summarize the key issues as they relate to the optimal solution obtainable using AQUARIUS. Those issues include the basic assumptions of the economic efficiency paradigm and the limitations of its practical application, as well as the key strengths of an emphasis on economic efficiency. We also review what is known about the demand for water in agriculture, municipalities, industry, hydroelectric energy generation, and recreation; and discuss options for specifying monthly demand curves.

Current Water Allocation Systems

Existing water allocations have developed under systems of water rights. In the United States those rights have centered around either the Riparian or the Prior Appropriation Doctrine. The Prior Appropriation Doctrine has been prevalent in drier regions of the U.S. where there is often insufficient water to meet all use requests and where careful modeling of water storage and allocation is most necessary. Under this doctrine, water is allocated according to a time-based priority rule whereby the water available to satisfy a new application is reduced by the sum of all prior established rights. Thus, only the remaining unappropriated flow is available to satisfy new applications. In some streams, late appropriations are left with little flow, especially in drier years.

During European settlement of the Western U.S., the primary emphasis for water use was on offstream uses, most importantly farming, mining, and municipal or rural domestic uses. Later, economic development enhanced the importance of water use by industry and the production of hydroelectric energy. As streams became more fully appropriated and heavily managed, and as urban populations and values grew in importance, instream flow for recreation and habitat became another important water use (Gillilan and Brown 1997).

Date of application for a water use does not necessarily indicate a use's value. In addition, the values of the water uses may change with shifts in technology, demographics, incomes, etc. Such shifts have been dramatic during this century, leading to significant changes in the economic value of water in different uses. A time-based priority system favors early arrivals, which is not an efficiency problem if unappropriated flow is ample or, in fully appropriated rivers, if voluntary transfers among water users are obstacle free. However, a time-based allocation in a heavily appropriated river can become inefficient if, as is commonly the case with water, institutional barriers or market failures impede the voluntary transfer of rights from lower-valued to higher-valued uses.

Regardless of the process by which the existing allocation of water in a basin was established, it may be helpful to compare that allocation with an efficient allocation because: 1) such a comparison may indicate promising opportunities for private water trades; 2) where trades are hampered or precluded by institutional barriers, the comparison may indicate areas where institutional reforms may allow for a more efficient water allocation; and 3) where past water developments were publicly financed, the comparison may indicate directions that the public entity should consider to increase the efficiency of the public project. Also, when considering a new publicly-financed water development, the opportunities for the project to increase the efficiency of water allocation should be carefully analyzed.

Economic Efficiency

Two fundamental economic concepts are efficiency and equity. Economic efficiency is concerned with the aggregate wealth generated in a society; economic efficiency is improved if aggregate wealth increases. Equity is concerned with the fairness of the distribution of that wealth; equity is improved if fairness increases. Economic efficiency is subject to technical evaluation, given the assumptions of the dominant (neoclassical) economic model. Equity is not subject to technical evaluation within the neoclassical model. Therefore, regarding equity, the role of economics within this theoretical model is to indicate the effect of a policy on income distribution so that equity can be considered in the political arena.

Three assumptions of the neoclassical economic model are particularly relevant. The first is that, for the purpose of evaluating economic efficiency, the existing income distribution is accepted as given. This is important if a different and more acceptable income distribution would result in different demand and supply schedules, and thus in different prices for goods and services. Acceptance of the existing income distribution is justifiable in a democracy because the income distribution is, to a large extent, a matter of public policy.

The second assumption is that consumers are the best judges of their welfare; thus, the values that individuals assign to goods and services are unquestioned. This consumer sovereignty assumption is not fully met, if for no other reason than that consumers often lack accurate information. However, in a democratic society, consumer sovereignty is not an unreasonable premise upon which to base resource allocation. Establishment of a democracy is, in a sense, a repudiation of allowing a select few to control resource flows.

The third assumption is that impacts outside the boundary of the analysis can be ignored. In water resource literature, the boundary of the analysis has been called the "accounting stance." The accounting stance separates the geographic or demographic area of concern from that which is ignored. This choice of boundary is important because it can determine whether a particular benefit or cost is relevant to the analysis. For example, effluent that lowers downstream water quality is ignored if the downstream area is beyond the boundary of the analysis. Typically, the accounting stance coincides with the administrative responsibility of the entity performing the analysis, but might be restricted by such things as data availability or the capacity of an analysis tool.

If benefits and costs occur at different points in time and the timing of their occurrence matters, the efficiency criterion must account for time. The traditional approach to this problem is to compute a present value (PV). An allocation of resources across the total number of time periods (*np*) is efficient if it maximizes the PV of net benefits that could be received from all the possible ways of allocating those resources over the *np* periods. Mathematically, this is accomplished by evaluating the discounted algebraic sum of a stream of benefits minus costs over the life of the project or period of analysis, as follows:

$$PV[r] = \sum_{i=0}^{np} \frac{(B_i - C_i)}{(1+r)^i} \tag{2.1}$$

where B_i is the benefit and C_i is the cost in period i, np indicates the total period of analysis, and r is the discount rate. Choice of the discount rate may depend on the opportunity cost of capital and can include a risk adjustment. Typically r is below 0.1. If the future will not be discounted, r is set to zero. The current version of AQUARIUS (V96) does not compute discounted benefits, although provisions have been made in the model for its future implementation. In what follows, we assume a zero discount rate.

It is important to recognize the distinction between economic and financial efficiency. Financial efficiency is concerned only with money flows. From this standpoint, a good or service is of value only if money changes hands when it is exchanged or consumed. Economic efficiency is concerned with all goods and services valued by the public regardless of whether consumption is accompanied by monetary exchange. Financial efficiency is improved if net financial return increases. Economic efficiency is improved if the net wealth of society increases. For example, additional instream recreation opportunities available without cost to users are ignored from the standpoint of financial efficiency but are relevant to economic efficiency if recreationists would be willing to pay for the opportunities if they had to pay to use them. Financial efficiency is a primary concern of private firms and corporations but may also be of interest to governments evaluating their cash flow for budgetary purposes. Economic efficiency is typically a primary concern of governments or organizations charged with administration of public welfare.

AQUARIUS can be used to evaluate either financial or economic efficiency. The key differences between evaluating financial and economic efficiency are which water uses are included and how the values of those uses are characterized. Financial efficiency may ignore some uses that are relevant to economic efficiency. The value of a water use to a business is often a constant return per unit reflecting a market or negotiated price, which is usually represented by a horizontal demand curve. The value of a water use to society may change depending on the amount of water use at issue and is usually represented by a sloped demand curve. In the following, we focus on the more complex situation of sloped demand curves.

Economic efficiency is maximized when the sum of benefits minus costs from the various outputs is as high as possible given people's values—as expressed by their willingness to pay or

willingness to accept compensation—and the physical constraints of the system (the production functions). In the remainder of this chapter, we assume that costs are nil and that water uses are independent (i.e., willingness to pay for a particular water use is independent of willingness to pay for other uses). We also assume a Marshallian demand function rather than the more precise Hicksian (income compensated) demand function. A Marshallian demand function will closely approximate its Hicksian equivalent if income effects are small (Willig 1976). Income effects should be small where the good at issue is a very small portion of the total purchases of the individual, as is generally the case with water.

Given these simplifications, P is price (i.e., marginal willingness to pay) and x is level of output in the following demand (i.e., marginal benefit) function: $P = f(x)$. With demand functions for the various water uses (j), the total benefit function (TB) to be maximized over the various time periods (i) is:

$$TB = \sum_{i=1}^{np} \sum_{j=1}^{nu} \int_0^{a_{ij}} f_{ij}(x_{ij})\, dx_{ij} \tag{2.2}$$

where: np = total number of time periods
nu = total number of water uses
a = level of consumption.

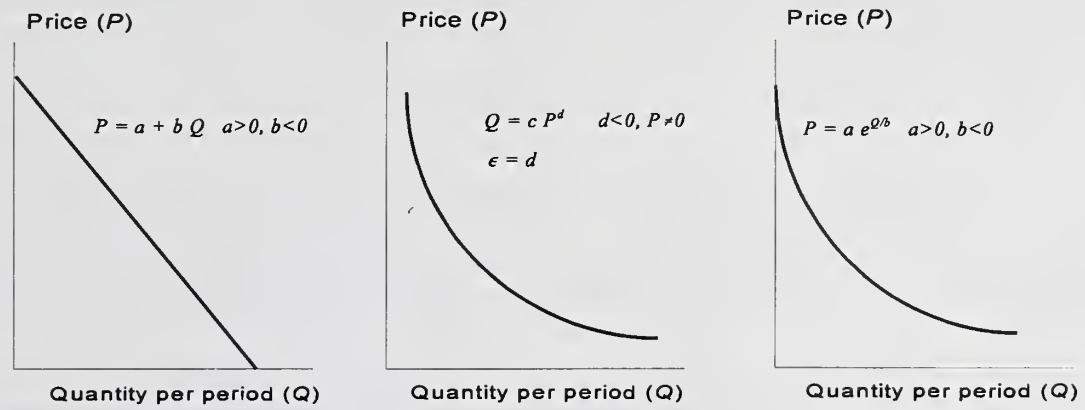
TB is maximized when a_{ij} are set so that the Ps are equal for all i and j. In other words, total benefits are maximized when the levels of consumption are such that the marginal benefits for each use across all uses and time periods are equal. TB can only be maximized over the j uses for which marginal benefit functions are specified. If relevant uses are omitted because their benefit functions cannot be specified, they must be represented in some other way such as by adding a physical constraint to the model.

Economic Demand Functions

In subsequent sections we report on past attempts to estimate demand functions for the various water uses included in AQUARIUS. In this section we review some general concepts.

Demand functions are not easily estimated from market data. At any one time, all that is observed is one point along the demand function, which is one price-quantity combination among a large number of combinations that could be observed under different conditions. However, by various techniques, including observation of different real-world markets containing similar people in different supply situations, economists have learned about demand functions. Typically they find that as the price decreases the quantity consumed increases, yielding a downward sloping demand function (figure 2.1). Further, although it is common in economics text books to illustrate demand functions using straight lines (figure 2.1a), most actual market demand

functions are probably nonlinear. Of course, short portions of any demand curve can be approximated by a straight line.



a) Linear demand curve b) Constant elasticity demand curve c) Exponential demand curve

Figure 2.1 Demand curve examples.

It is common to describe a point along a demand function in terms of its price elasticity of demand (hereafter just "elasticity" or ϵ), which expresses the percentage change in quantity demanded for a percentage change in price. At a point along the curve, $\epsilon = \partial Q/Q \;/\; \partial P/P$ where Q is quantity demanded and P is price. For a downward sloping curve, an $\epsilon < -1$ is "elastic," and an $\epsilon > -1$ is "inelastic." The special case of $\epsilon = -1$ is "unit elasticity." A linear downward sloping demand curve is elastic in the upper left and inelastic in the lower right, with a point of unit elasticity in between. A constant elasticity downward sloping demand curve is convex to the origin (figure 2.1b), and undefined at a zero price. Such a curve is represented by a power function such as $Q = c\,P^d$ with $c > 0$ and $d < 0$. The exponent on price d is equal to ϵ.

A demand curve in power form can be estimated by knowing ϵ and one price-quantity point along the curve. Because the curve does not cross either axis, the ends of the curve may be very sensitive to values of the particular point specified. An attractive alternative to the power function is the exponential function $Q = g\,e^{P/h}$, with $g > 0$ and $h < 0$. Note that in AQUARIUS, when specifying exponential functions, we have adopted the convention of expressing the coefficient in the exponent in its reciprocal form. This avoids strings of 0s when the exponent is very small, as it often is. Also, in AQUARIUS the minus sign is assumed when the user enters the coefficient in the exponent. An exponential function can approximate a power function and has the additional feature that the curve crosses an axis. The demand function $Q = g\,e^{P/h}$ allows the demand curve to cross the quantity axis, indicating that satiation is reached. The inverse demand function $P = a\,e^{Q/b}$ with $a > 0$ and $b < 0$, allows the curve to intercept the price axis indicating that consumption ceases if the price is high enough (figure 2.1c). Elasticity varies along an exponential function. The exponential function can be estimated by knowing two points

along the curve. One point that may be known, or at least more easily assumed than others, is the price at which quantity demanded falls to zero. Call this price $P_{Q=0}$. Another convenient point is given by the quantity at which price equals the existing price P_e, call it $Q_{P=Pe}$. Given these two points, the intercept $a = P_{Q=0}$ and the exponent $b = Q_{P=Pe}/[ln(P_e) - ln(P_{Q=0})]$ of the inverse demand function $P = a\, e^{Q/b}$.

The two-point approach to estimating an exponential function fails to use information about elasticity of demand that may be available for at least one point along the demand curve. AQUARIUS includes a computational tool to estimate the coefficients of the exponential function knowing two points and the elasticity at one of those two points. The tool uses a weighted least-square method to minimize the sum of squares of deviations of the observed points and elasticity from those of the fitted function. See Appendix B for details.

It is important to distinguish between demand for water in general and demand for water from the particular source being modeled. For example, demand for the first increments of residential water is extremely high. Thus, a demand curve for residential water in general might intersect the price axis at a very high price. However, residential water from a particular source may have alternative, though more costly, sources. The cost of the next most costly source would truncate the demand curve for the particular source at issue at a price equal to that cost. Whenever an alternative source exists, its cost can serve as an estimate of $P_{Q=0}$.

When comparing demand functions for water in different uses it is important that each demand function be for raw water. For example, it would be incorrect to compare demand for raw water in irrigation with demand for treated and delivered water in residential use. The latter estimates of willingness to pay would include the cost of treatment and delivery whereas the former would not. If the process of estimating a demand function for a particular water use focuses on treated and delivered water, treatment and delivery costs must be subtracted from the estimated willingness to pay to yield demand for raw water.

For offstream water uses that do not completely consume the diverted water, such as most agricultural and municipal withdrawals, the quantity variable in a water demand function must be chosen to correspond with the way diversions are being modeled. Sometimes in modeling water allocation, only the consumptive use amount is modeled; that is, withdrawal is set equal to the consumptive use and return flow is thereby ignored. This is a viable procedure if actual return flows reach the stream upstream of the next downstream diversion point of the model. In this case, consumptive use is the appropriate quantity variable for the water demand function. A more accurate characterization of water movement in a river basin is achieved by explicitly modeling return flows and specifying the point where they reach the water course. In this case, withdrawal is the appropriate quantity variable. Either way, consumptive use includes actual evapotranspiration at the location of use plus any other losses along the delivery and return paths.

In a well functioning water market, the marginal value of raw water is equal across uses. This equality occurs because, if marginal values differ, the higher-valued use can afford to purchase

water from a lower-valued use, paying a price that exceeds the water's value in the lower-valued use. Transfers from lower-valued to higher-valued uses continue until the advantages of trade are eliminated, that is, until marginal values are equal and an optimal allocation is reached. Equality of value at the margin does not indicate equality of value over the full extent of the demand curve. Indeed, the value of the initial units of water to residences or the industry is greater than it is to irrigated agriculture. However, in the absence of institutional barriers to trade or significant transaction costs, actual marginal raw water deliveries to each water use will occur at the point of equal marginal value.

Markets in the real world rarely perform as smoothly as they do in theory. Water transfers are often constrained by institutional barriers such as contractual agreements attached to publicly funded storage and delivery projects that preclude sale to unauthorized uses, or by the lack of legal recognition of some water uses such as habitat maintenance. Water transfers are also typically impeded by significant transaction costs such as those associated with quantifying and legally defending return flow amounts. Institutional barriers and transaction costs may keep marginal values of water in different uses from reaching equality, requiring separate valuation efforts to estimate the current marginal values of water in different uses.

In the following sections, we present information that will facilitate use of AQUARIUS including: 1) evidence on the shape of the annual demand curve; 2) detail on placing the annual demand curve in realistic price-quantity space; 3) information on the difference between demand for treated and delivered water and demand for raw water; and 4) information on disaggregating annual demand to the monthly time step. Prices from studies performed over the past twenty years or so are reported. We have, unless stated otherwise, adjusted for inflation by updating these prices to 1995 dollars using the Gross Domestic Product (GDP) implicit price deflator.

Irrigation Water Use

Irrigation in the United States accounts for 42 percent of fresh water withdrawals and 83 percent of consumptive use (table 2.1). In the 12 most western contiguous states, 82 percent of withdrawals and 92 percent of consumptive use is for agriculture. In some states, such as Colorado, irrigation plays an even more important role (table 2.1).

The most direct approach to estimating the demand for irrigation water is to observe market transactions where water rights are purchased by farmers in a competitive water market. Such transactions, however, seldom occur. Although farmers in some parts of the West commonly sell water rights to municipal and industrial water users, and sometimes do so in relatively competitive markets, instances of open market water purchase by farmers are rare.

Lacking a direct measure of farmers' demand for irrigation water, economists have focused on water's role as an agricultural input. This role affects two end products that are commonly sold in competitive markets; farm land and farm produce. Thus, water demand schedules are derived by

estimating the contribution of irrigation water towards either the value of irrigated farms or the value of farm produce. The viability of these approaches rests on the degree to which the end products and inputs, other than raw water, are competitively priced. Subsidies and other impediments to the free working of markets can alter prices to such an extent that the estimated water price reflects market manipulation or control rather than resource demand and scarcity. Although the competitive market requirements are rarely met completely, they have been met to a sufficient degree to sustain, over the past 40 years or so, numerous studies of the value irrigation water.

Table 2.1 Water use in the United States in 1985 for the major consumptive use types in millions of gallons per day, with percent of total use in parentheses.[a]

	United States		Western States[b]		Colorado	
			Withdrawal			
Residential	24,300	(7)	8,813	(7)	743	(4)
Commercial	6,940	(2)	2,068	(2)	120	(1)
Industrial	28,100	(9)	3,958	(3)	128	(1)
Thermoelectric	131,000	(40)	8,840	(7)	123	(1)
Irrigation	137,000	(42)	111,610	(82)	12,400	(94)
			Consumptive Use			
Residential	5,680	(6)	2,714	(5)	145	(3)
Commercial	1,190	(1)	501	(1)	21	(0)
Industrial	4,200	(5)	1,083	(2)	28	(1)
Thermoelectric	4,350	(5)	525	(1)	37	(1)
Irrigation	73,800	(83)	55,090	(92)	4,570	(95)

[a] Source: Solley et al. (1988). Fresh water only, surface plus ground water. Ignores mining and livestock water use. Percent columns may not add to 100 because of independent rounding.

[b] Includes Arizona, California, Colorado, Idaho, Montana, Nevada, New Mexico, Oregon, Texas, Utah, Washington, and Wyoming.

Shape of the Annual Demand Curve for Irrigation Water

Because irrigation water attached to farm land contributes to the land's value, multivariate statistical methods can be used to isolate that contribution. This "hedonic" or "property value" method requires a data set of farm sales that includes sale price and all the characteristics that contribute to the farms' value. The coefficient on the irrigation water availability characteristic indicates the water's average per-farm value. Early studies using the hedonic method to value

irrigation water include Milliman (1959), Hartman and Anderson (1962), and Knetsch (1964). Torell et al. (1990) used this approach with data from over 7,000 farm sales in five Western states to compute the present value of the average acre-foot of ground-water available for irrigation, which was about $5 for the study area as a whole. This approach yields only a point estimate of the value of water, not an entire demand curve.

More common are studies that estimate the contribution of irrigation water to farm produce. Three approaches have been used, the: 1) production approach, 2) programming approach, and 3) econometric approach. The production approach involves observing the effect of irrigation on the production of specific crops. This approach has two variants. One uses data from working farms and involves comparing production on irrigated fields with production on nonirrigated fields, where other inputs—soils, rainfall, management, etc.—are roughly the same on the two sets of fields. This approach yields a point estimate of the value of irrigation water. For an example, see Duffield et al. (1992, p. 2179). The other variant involves controlled studies at experimental farms where the effect of water on the production of specific crops is carefully observed. Two such studies are Ayer and Hoyt (1981) in Arizona and Kelly and Ayer (1982) in California. These studies estimate production functions expressing the input-output relation between water and crop production and then infer the value of the water input from the market value of the crop. Both studies reported that water demand was inelastic for most crops studied over water prices as high as $100/acre-ft (most elasticities were greater than –0.5). However, such studies are limited by the combinations of other inputs that the experimental plots were able to incorporate, allowing for the possibility of more elastic demand in the long-term (i.e., when all inputs can be adjusted with more flexibility than they were at the experimental plots). This approach yields demand estimates for individual crops, not for entire multi-crop farms or for multi-farm regions.

With the programming approach, farmers' production options and input and output prices are expressed in matrices of linear or quadratic problems. Optimal production decisions are determined via solution of the corresponding programming problems. This approach is typically applied to individual representative farms. The results of such runs may be extrapolated to model efficient allocations across all the farm land in a given agricultural region. Water demand functions are typically estimated using this approach by solving for irrigation water input under different assumed water prices (yielding estimates of quantity demanded) or assumed water availability (yielding estimates of marginal water value). With enough different solutions, a water demand curve is plotted giving the maximum amount farmers in the region could pay for increments in irrigation water.

Some studies using the programming approach have presented stepped demand functions showing the values of discrete quantities of water that would be used by a farm or by all the farms in the given agricultural region. Each step along the curve represents a different crop or the same crop in a different situation (e.g., on a different soil). Typically these stepped demand schedules indicate a nonlinear water demand function, one stepper in the upper left than in the lower right (Anderson et al. 1973, Bernardo et al. 1987, Bowen and Young 1985, Kelso et al. 1973, and Kulshreshtha and Tewari 1991), but sometimes they show a stepped function that

approximates a linear curve (Gisser 1970, Taylor and Young 1995). Figure 2.2 is an example of a stepped demand function. This function is more accurately expressed as an exponential function than as a power function. A regression yields the following inverse demand function: $P = 140\ e^{-Q/166,667}$ ($r^2 = 0.63$). The corresponding power function has an r^2 of 0.44.

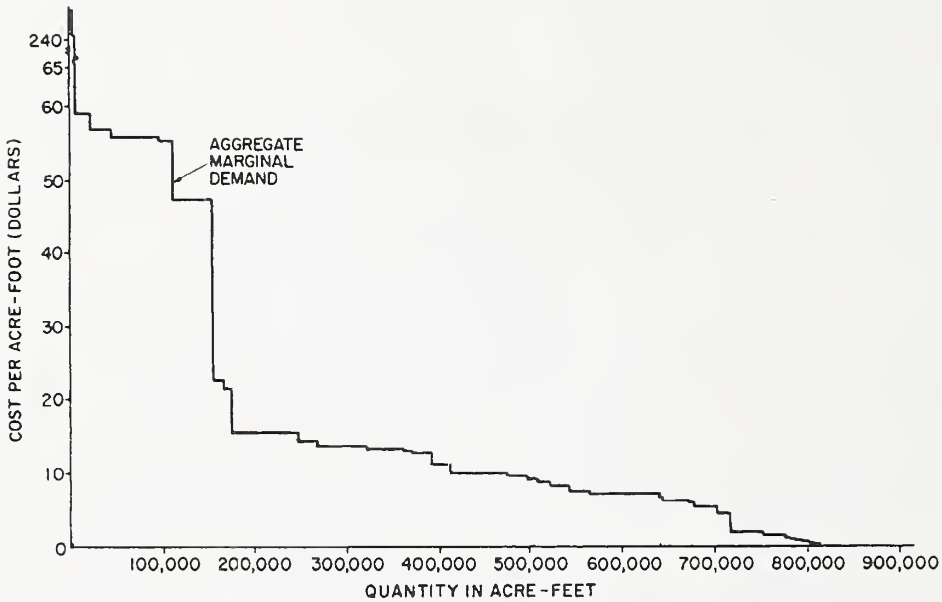


Figure 2.2 Derived marginal demand for irrigation water (from Kelso et al. 1973, figure 4-11).

The convex shape (to the origin) of the demand function estimated by many programming studies reflects the fact that feed grain and forage crops typically occupy most of the irrigated acreage and yield less profit per acre-foot of water applied than do the more highly valued, small acreage, fruit and vegetable crops. Thus, the feed grain and forage crops occupy the lower right portion of the demand curve and the fruit and vegetable crops occupy the upper left portion. The exact shape of the curve in a given area depends on which crops are planted in that area, which will in turn depend on physical variables, like the weather and soil, and on prices of inputs and outputs.

Other programming studies plot smooth curves through the price-quantity points determined in model solutions or report equations that represent such points. For example, Heady et al. (1973) used a large scale interregional model of U.S. agriculture to derive a linear demand curve for irrigation in the Western U.S. The model allowed for substitution of dry land farming for irrigated farming as the price of irrigation water was increased. Howitt et al. (1980) studied irrigation in California and produced discrete price-quantity points that very closely traced out a demand curve convex to the origin. Vaux and Howitt (1984) later used this curve to estimate demand functions for irrigation in northern and southern California expressed in the form: $Q = j + k\,P^{0.5}$, where Q = quantity and P = price, with $j > 0$ and $k < 0$. In contrast, based on the

work of Moore et al. (1974), Vaux and Howitt (1984) adopted a downward sloping linear demand curve for irrigation water demand in California's Imperial Valley. Recently, Booker and Young (1994) adapted earlier programming studies of agriculture in the Upper Colorado River Basin to express agricultural profit as a second degree polynomial of water input that, when used to derive water demand, yields a demand function accurately represented by the power function $Q = c P^{-0.66}$, with $c > 0$. The Vaux and Howitt (1984) and Booker and Young (1994) studies both showed downward sloping demand functions that were convex to the origin.

The econometric approach has been applied with cross-sectional data representing a large geographical area over which water costs, perhaps because of pumping depths or application rates, vary substantially. For example, studies by Moore et al. (1994), Ogg and Gollehon (1989), and Frank and Beattie (1979) each used data from several Western U.S. states. Ogg and Gollehon regressed water use on water price variables using three different functional forms. The most successful model used a power function, yielding an irrigation water demand function that is convex to the origin, with a constant elasticity of –0.26. Frank and Beattie assumed a power function and regressed the value of agricultural production on a set of explanatory variables. They found elasticities of demand for irrigation water from –1.0 to –1.5 depending on the geographical region. Using data from seven Texas counties, Nieswiadomy (1985) regressed groundwater pumping on explanatory variables including pumping cost and compared linear and log-log (i.e., power) functional forms. The functional forms differed little in their ability to explain variance in the pumping rate. The log-log coefficient on pump cost, equivalent to the elasticity of demand, was –0.8.

All past studies of demand for irrigation water found a downward sloping demand curve. Nearly all programming studies found a nonlinear demand curve, one convex to the origin. These curves were inelastic, with ϵ generally greater than –0.3 at low water prices and elastic at high water prices (table 2.2), and can be approximated by an exponential function. Of the studies that fitted smooth curves to these results, Vaux and Howitt (1984) used nonlinear and linear functions and Booker and Young (1994) presented a profit function that yielded a demand function that we most accurately expressed in an equation in power form. The econometric studies generally either assume a power function—a common practice in economic studies partly because the exponent on price in a demand function is equal to the price elasticity—or find upon comparison that such a function performs better in terms of explained variance.

Table 2.2 Characteristics of estimated multi-crop water demand functions for the Western United States.

Author	Location	Method	Elasticity [a] Min [b]	Max [b]
Anderson et al. (1973)	Utah	Programming	-0.01	-7.0
Bernardo et al. (1987)	Washington	Programming	-0.14	-0.4
Booker & Young (1994)	Colorado	Programming	-0.6	-0.6
Frank & Beattie (1979)	Western U.S.	Econometric	-1.1	-1.1
Gisser (1970)	New Mexico	Programming	-0.17	-3.0
Heady et al. (1973)	Western U.S.	Programming	-0.36	-1.5
Howitt et al. (1980)	California	Programming	-0.46	-1.5
Kelso et al. (1973)	Arizona	Programming	-0.09	-1.2
Kulshreshtha & Tewari (1991)	Saskatchewan	Programming	-0.05	-3.1
Ogg et al. (1989)	Western U.S.	Econometric	-0.26	-0.26
Vaux & Howitt (1984)	California	Programming	-0.1	-10.

[a] Elasticities are arc or point and may be as reported in the original paper or computed from information presented in the paper. Entries are approximate and may represent a compromise between several estimates presented in the original paper.
[b] The minimum and maximum values of irrigation water reported for the estimated demand function.

Representative Annual Irrigation Water Demand Curve

The amount of irrigation water diverted in a given farming area depends on many factors. If water is not limited it will be diverted up to the point where its marginal value equals the farmer's marginal cost of acquiring and applying the water. Where water limitations keep irrigation below the level at which marginal value equals marginal cost, the demand curve is truncated by the resource constraint. In any case, as the price of water increases, water use per acre may drop somewhat, but the main change will probably be a reduction in the number of acres irrigated as acres are converted to dry land crops or removed from production. Therefore, the quantity axis of the demand curve cannot be expressed on a quantity per-acre basis, unlike the demand curve for residential water, which can be expressed on a quantity per-capita basis, as every person needs some water. The appropriate variable for the quantity axis is water volume per year for the entire agricultural area.

Ideally, the demand curve for the agricultural area(s) to be included in a river basin water allocation model should be carefully derived. However, where a demand curve has not been derived, an approximation can be achieved by adopting a functional form and knowing one or two points along the curve. Assuming an exponential form, two points are needed such as $P_{Q=0}$ and $Q_{P=Pc}$. To provide rough estimates of these points, various papers were examined. The prices at the upper left of the demand curves (approximating $P_{Q=0}$) varied from under \$100 to over

\$200/acre-ft. The estimated maximum prices depend on whether high-value fruits and vegetables are grown in the area and whether the study included those crops when estimating the demand curve. The quantity demanded at the existing marginal price ($Q_{P=Pe}$) will vary with the acreage available for planting, soil type, weather (principally temperature and precipitation), method of application (flood versus spray), etc. Existing prices are typically from \$5 to \$15/acre-ft for surface water delivered via gravity but may vary more widely if pumped. The following studies reported ground water pumping costs: Ogg and Gollehon (1989) described costs from \$54 to \$102/acre-ft across the West; Moore et al. (1994) reported costs ranging from \$25 to \$32 across the West; and Nieswiadomy (1985) related a cost of about \$7 for areas of Texas. These prices should correspond to withdrawals per acre across the West that, according to Solley et al. (1988, table 7), vary from 1.22 acre-ft on average in the Texas-Gulf region to 5.54 acre-ft on average in the Upper Colorado region. For comparison, the weighted mean fresh water agricultural withdrawal over the nine Western water resource regions was 2.94 acre-ft/acre (Solley et al. 1988).

Because there is considerable variety across the West, some knowledge of local conditions is needed to estimate $P_{Q=0}$, P_e and $Q_{P=Pe}$. But to provide an example, assume an exponential functional form, where $P_{Q=0}$ = \$150, P_e = \$15, an irrigated area of 10,000 acres, and existing annual withdrawals of 3 acre-ft/acre. Using these assumptions, the annual inverse demand function is $P = 150\ e^{-Q/13,000}$.

Demand for Raw Water

The cost of irrigating crops includes the expense of delivering water to the farm and applying it to the crops. Demand for raw water is estimated net of delivery and application costs. One approach to estimating the demand curve for raw water is to subtract delivery and application costs from willingness to pay for delivered and applied water.

Delivery costs for gravity fed water are mainly the expense of constructing, maintaining, and operating dams and canals. In most gravity fed irrigation areas, these functions are performed by an association, such as an irrigation district, which charges individual farmers a fixed assessment fee per unit of water owned to cover the association's costs. Thus, the assessment fee approximates the amortized average cost of storing and delivering the water. Such fees typically vary from \$5 to \$15/acre-ft. (A further complication, when evaluating economic efficiency, is the effect of government subsidies on the values of agricultural products and the costs of agricultural inputs. Many of the West's dams and canal systems have been publicly subsidized to some extent.) Delivery costs for pumped water are largely the pumping expense. Marginal pumping cost is a function of energy cost, which is typically constant for an individual farmer.

Application costs vary by method. These costs may include fixed costs, such as for field leveling, installing ditches, and installing sprinkler equipment, and variable costs such as for hand moving of syphons or sprinkler heads and maintaining ditches or sprinkler equipment.

Monthly Distribution of Annual Demand

Farmers make water use decisions at various times. In the long run, all factors of production under the farmer's control are variable. For example, the choice of whether to install a sprinkle irrigation system is variable in the long run. Annual decisions (intermediate run) focus largely on what to plant given expected prices and water availability. Monthly decisions (short run) usually focus on responding to recent changes in prices and weather patterns. Daily decisions (very short run) might focus on how much water to apply to a specific field given existing moisture conditions. Decisions about the purchase of water rights or drilling a well are long or intermediate run decisions; decisions about rental of a water right (i.e., purchase of water for one-time use) are intermediate- or short-run decisions; and decisions about when to irrigate are short-run or very short-run decisions.

Farmers have more flexibility to make input substitutions in the long run than in the short run. Thus, responsiveness to water price decreases as the planning horizon shortens. The demand curve for water depends partly on the degree of flexibility to make substitutions; thus, demand curves may vary with length of run. Most studies of agricultural water demand mentioned above used methods and data corresponding to intermediate- or long-run planning, so their results apply to an annual or longer planning horizon. We are not aware of any studies that estimate monthly demand curves. So the question becomes, how to disaggregate an annual demand curve to the monthly time step.

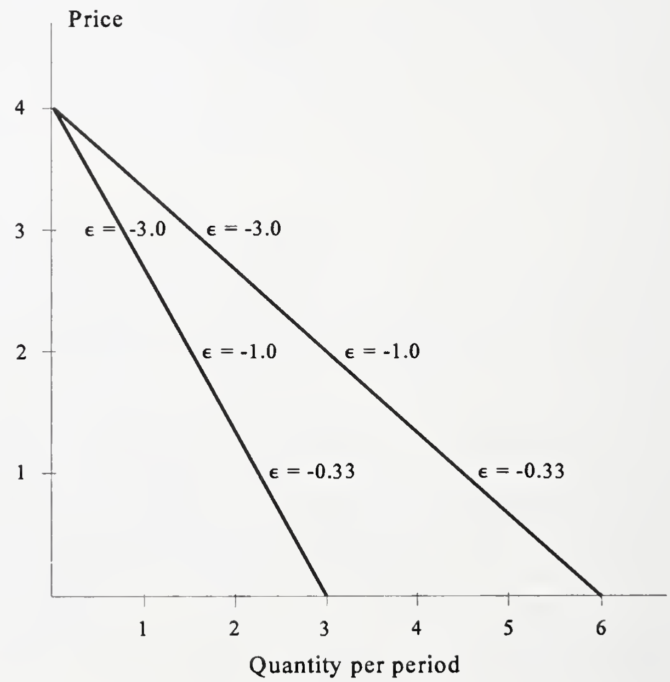


Figure 2.3 Disaggregation of an annual demand curve.

Perhaps the most direct approach is to disaggregate horizontally the annual quantity demanded among the irrigation months following the typical monthly distribution of annual water needs for the area. For example, assume a linear annual demand curve for water as depicted by the right-hand line in figure 2.3, in a region where irrigation occurs during only two months, which each receive exactly half of the annual irrigation amount. In this case, the left-hand line depicts demand for each of the two months.

Table 2.3 shows the monthly breakdowns of annual irrigation for two large basins in the West. The monthly distribution of annual irrigation demand depends largely on the weather and on whether double cropping is practiced, which also depends on the weather. As table 2.3 indicates, irrigated farming occurs year-round in the Lower Colorado Basin, which includes the desert areas of Arizona and California but not the cooler Upper Basin. A limitation of this approach is that it holds the maximum price, which is the intersection at the price axis, constant for all months at the level set by the annual demand curve, when in fact the farmer might be willing to pay more or less in some months. For example, because in a given month flexibility is limited to substitute among inputs, a farmer might be willing to pay more for initial water quantities than the annual demand curve indicates. Another limitation is that the assumption of constant monthly proportions of annual water use precludes the possibility of adjusting the use among months in response to existing moisture conditions. Nevertheless, given the lack of estimated monthly demand functions and of data on how irrigation responds to natural moisture changes, the constant proportion disaggregation approach is probably the most viable.

Table 2.3 Monthly allocation of annual irrigation withdrawals (in percent).[a]

	Colorado Basin[b]	
	Upper	Lower
January	0	4
February	0	7
March	0	9
April	4	10
May	23	10
June	35	11
July	23	12
August	12	11
September	3	9
October	0	7
November	0	5
December	0	4

[a] Source: U.S. Bureau of Reclamation (1986).
[b] Columns may not sum to 100 because of rounding.

Municipal Water Use

Distinctions among types of municipal and industrial (M&I) uses are unclear in the economic literature. Some studies of "residential" or "domestic" demand use data from a sample of individual households and are clearly residential demand studies. Others use aggregated data from water providers that include deliveries to commercial, and possibly some industrial users, although the bulk of the deliveries are to residential users. In these studies, the terms "community," "municipal," and "residential" are used inconsistently. In the following, "residential" and "municipal" studies are together except when the "municipal" studies present separate estimates for nonresidential users. Commercial use is presented separately. Industrial use is covered in the next section.

Residential water use in the U.S. is 7 percent of the total water withdrawal; commercial water use is an additional 2 percent. In relatively dry states with large agricultural sectors, such as Colorado, these percentages are even lower (table 2.1). Although small in terms of quantity, municipal water use looms large in terms of average value per unit.

Shape of the Annual Demand Curve for Residential Water

Residential water demand has received considerable study. Danielson (1979), Young et al. (1983), and Gibbons (1986) each summarized the price elasticity findings of earlier studies and examined 23 unique studies published by 1980. Tables 2.4 and 2.5 list the elasticity findings of another 16 residential water demand studies published since 1980. All estimates of elasticity from these 39 studies are negative, indicating the expected downward sloping demand curve. Furthermore, nearly all estimates are above –1, indicating inelastic demand. On an annual basis, estimates of elasticity of demand among the 39 studies vary from –0.02 to –1.24, with most from –0.3 to –0.7.

Table 2.4 Price elasticity of annual demand for residential water.[a]

Author	Location	Elasticity
Billings (1990)	Tucson, AZ	-0.57
Griffin (1990)	Texas	-0.37
Hanke & de Mare (1984)	Malmo, Sweden	-0.15
Jones & Morris (1984)	Denver, CO	-0.14 to -0.44
Martin & Thomas (1986)	5 cities in arid areas	-0.49
Nieswiadomy (1992)	Northeastern U.S.	-0.28
	Southern U.S.	-0.60
	Western U.S.	-0.45
Rizaiza (1991)	Saudi Arabia (4 cities)	-0.40, -0.78
Schneider et al. (1991)	Columbus, OH	-0.26 to -0.50
Stevens et al. (1992)	Massachusetts	-0.41 to -0.69
Williams & Suh (1986)	United States	-0.49

[a] In most cases, the elasticities were estimated from a constant-elasticity functional form. Where a variable-elasticity functional form was used, we report the arc or point elasticity that was reported in the original, which is estimated at the average price for the sample. A range is also listed if different estimation procedures or different conditions resulted in more than one estimate. Where both long- and short-run elasticities were reported, only the long-run elasticities are listed. Griffin (1990), Hanke and de Mare (1984), and the -0.5 estimate of Schneider et al. (1990) were listed as "municipal;" all others were listed as "residential."

Table 2.5 Price elasticity of seasonal demand for residential water.[a]

		Elasticity	
Author	Location	Winter	Summer
Danielson (1979)	Raleigh, NC	-0.31	-1.38
Griffin (1990)	Texas	-0.32	-0.40
Griffin & Chang (1990)	Texas	-0.16	-0.38
Grima (1973)	Toronto, ON	-0.75	-1.07
Howe (1982)	U.S. (21 cities)	-0.06	-0.43 to -0.57
Lyman (1992)	Moscow, ID	-0.65	-3.33
Renzetti (1992)	Vancouver, BC	-0.01	-0.65

[a] Griffin (1990) and Griffin and Chang (1990) were listed as "municipal" and all others were listed as "residential."

Studies (e.g., Schneider et al. 1991, Lyman 1992) show that long-run demand is more elastic than short-run demand, which reflects adaptations to changing prices that occur gradually as consumers change habits and alter water using appliances or landscape vegetation. Most studies of water demand elasticity, however, have not attempted to measure both short- and long-run elasticities, leaving the length of run to be determined by their methods and data. The estimates of elasticity among the 39 studies include a mixture of short- to long-run estimates.

Nearly all studies of residential or municipal water demand use an econometric approach, most with cross-sectional data but some with time series data. Most studies assume a non-linear functional form, thereby predetermining the shape of the annual demand curve. Many studies assume a Cobb-Douglas (power) function that yields a constant elasticity curve; note the number of studies listed in table 2.4 with only one elasticity estimate, which indicates a constant elasticity model. Some other studies (e.g., Foster and Beattie 1979) assume an exponential function on water price.

Studies of water demand in the U.S., whether cross-sectional or time-series studies, have used data with relatively little variation in water price. For example, average prices charged by the 218 cities studied by Foster and Beattie (1979) varied across cities from \$0.10 to \$1.79/m^3, and average prices charged by the 30 Texas communities studied by Griffin and Chang (1990) varied across communities from \$0.34 to \$1.62/m^3. U.S. studies do not include cases with sufficiently high water prices to allow accurate estimates of the shape of the upper left portion of the demand curve. Thus, the estimates of price elasticity reported in the 39 studies apply to the low-price portion of the residential demand curve.

Martin and Thomas (1986) provided a unique estimate of the entire demand curve for residential water via their comparison of water use in five cities that, although sharing an arid climate, differ markedly in the cost of providing water and in the price charged to consumers. These prices

varied from \$0.16 to \$25.01/m^3 and represent cities in the U.S., Kuwait, and Australia (table 2.6). The authors reported that a demand curve with constant elasticity of –0.49 provided the best fit of the five points. We found one other high-priced case in the literature (Taif, Saudi Arabia) reported by Rizaiza (1991) (table 2.6).

Table 2.6 Residential water price and quantity data for selected cities.

Author	Location	Quantity (l/c/d)[a]	Price (\$/m^3)[b]
Martin & Thomas (1986)[c]	Coober Pedy, Australia[d]	50	25.01
	Kuwait urban areas	184	1.19
	Perth, Australia	288	0.55
	Tucson, AZ	371	0.48
	Phoenix, AZ	595	0.16
Rizaiza (1991)[e]	Taif, Saudi Arabia	35	8.39

[a] Liters per capita per day.
[b] 1995 dollars.
[c] Marginal prices.
[d] Desalinized water delivered by tanker.
[e] Average prices. Delivered by tanker.

Representative Annual Demand Curve for Residential Water

All evidence about residential demand suggests a downward sloping demand curve concave to the origin. Power and exponential functions are options for characterizing such a curve. An exponential function has the advantage of being able to intersect the price axis to represent the cost of a more expensive alternative if one exists. If the source being modeled is the municipal supplier, possible alternatives include trucking water in from other locations and purchasing bottled water from the supper market. For example, two of the locations listed in table 2.6 have prices for water delivered by tanker (one is for desalinized water). Bottled water sells for about \$1/gal (\$264/m^3). See Abdalla et al. (1992, table 1) and Laughland et al. (1993, table 2) for estimates of bottled water prices for some Pennsylvania communities.

One approach to estimating a representative residential water demand curve is to adopt an exponential function. The point of intersection along the price axis ($P_{Q=0}$) would equal the cost of the next more expensive source of residential supply. As seen above, this cost might range from \$25 to \$250/m^3; the exact cost would depend on the available alternatives in the location of interest. Assume, for the purpose of the example, that $P_{Q=0}$ = \$100. As an estimate of $Q_{P=P_e}$, the quantity demanded at the existing price P_e, we adopt the U.S. average. Americans paid an average of \$0.48/m^3 for publicly supplied treated domestic water and consumed an average of 397 liters per capita per day (l/c/d) (table 2.7). This yields an inverse demand function of P=100 $e^{-Q/74}$ using the exponential function approach described previously. A more accurate estimate of the curve might be obtained by incorporating elasticity information using the approach described in Appendix B, Case III.

Table 2.7 Residential water price and quantity data for Western states.[a]

State	Quantity	Price (\$/m^3)
Arizona	550	0.69
California	504	0.55
Colorado	572	0.38
Idaho	1076	0.33
Montana	557	n.a.
Nevada	812	0.34
New Mexico	678	0.64
Oregon	481	0.33
Texas	542	0.47
Utah	826	0.21
Washington	553	0.47
Wyoming	701	0.52
Western average[b]	654	0.45
U.S. (all 50 states)	397	0.48

[a] Domestic use of publicly supplied water in 1985 from Solley et al. (1988). Average cost of publicly supplied residential water, updated to 1995 dollars, from van der Leeden et al. (1990).
[b] Simple average for the 12 states listed above.

Commercial Water Use

Few studies have estimated demand for commercial water uses. Table 2.8 lists three of the available studies. One of these studies, by Schnieder et al. (1991), separated government and school use from other commercial uses. Schneider et al. also estimated residential demand and concluded that commercial demand was much more elastic than residential. This conclusion is sensitive to the particular kinds of commercial establishments included in their data. Lynne et al. (1978) estimated demand for numerous types of commercial establishments (e.g., motels and department stores) and found a wide range in price elasticity at existing water prices.

Given the small number of commercial water-use studies, the wide range in elasticity estimates among these studies, and the small variability in water price across studies, estimating a representative demand curve for commercial water is risky. However, it may be possible to use the residential demand curve to represent M&I uses, with an adjustment for the additional use per capita. As seen in table 2.1, commercial use is about 30 percent of residential use.

Table 2.8 Price elasticity of annual demand for commercial and industrial water.

Author	Location	Type of water use	Elasticity
	Commercial		
Lynne et al. (1978)	Miami, FL	Commercial	-0.11 to -1.33
Schneider et al. (1991)	Columbus, OH	Commercial	-0.92
		Government	-0.78
		School	-0.96
Williams & Suh (1986)	United States	Commercial	-0.36
	Industrial		
De Rooy (1974)	New Jersey	Industrial	-0.35 to -0.89
Elliott (1973)	29 US cities	Industrial	-0.64
Grebenstein & Field (1979)	United States	Industrial	-0.33 to -0.80
Renzetti (1992a)	Vancouver, BC	Industrial	-1.91
Renzetti (1992b)	Canada	Industrial	-0.15 to -0.59
Renzetti (1993)	Canada	Industrial	-0.66 to -2.17
Schneider et al. (1991)	Columbus, OH	Industrial	-0.44
Turnovsky (1969)	16 towns in MA	Industrial	-0.50
Williams & Suh (1986)	United States	Industrial	-0.74

Demand for Raw Water

The cost of treatment and delivery must be subtracted from the total payment to yield the raw water value. The best generally available estimate of treatment and delivery costs is the average cost of delivered municipal water because utilities providing water typically set rates to just cover their costs. As mentioned, the average U.S. cost is \$0.48/m^3 for publicly supplied treated domestic water, but costs may vary considerably from one location to another (see table 2.7 for state averages). Assuming constant returns to scale, the cost is subtracted from the willingness to pay over the full range of quantity demanded to yield the demand curve for raw municipal water.

Monthly Distribution of Municipal Demand

Several studies found that residential demand elasticity differs by season, with elasticity being greater in summer than in winter (table 2.5). The principal difference between the seasons is that outdoor use, largely for lawn and garden sprinkling, occurs in the warmer months. Apparently demand for outdoor use is more sensitive to water price than is demand for indoor use. However, whereas winter elasticities are consistently below summer elasticities, there is little consistency across studies in the level of elasticity in either season. Winter estimates range from –0.01 to –0.75 and summer estimates range from –0.38 to –3.33 (table 2.5). The wide range in elasticity estimates for a given season reflects climatic differences among the locations studied and methodological differences among the studies.

Indoor water use remains quite constant throughout the year, but outdoor use responds to weather. Table 2.9 gives monthly breakdowns of annual water use for some comunities in Texas and along the Colorado Front Range. The greater concentration of deliveries during the summer months along the Front Range versus Texas reflects the drier conditions and greater seasonal variability in weather along the Front Range.

As with irrigated agriculture, the most direct approach to specifying monthly demand curves is to horizontally disaggregate annual water demand based on the typical monthly proportions of annual water use. Because indoor water use is the most valuable portion of domestic use and does not vary with weather, the point where the demand curve intersects the vertical axis should remain constant across months. The greater elasticity of demand in the summer is reflected in the flatter curves of these large quantity months (figure 2.3).

Table 2.9 Examples of monthly allocation of of annual municipal deliveries (in percent)[a]

	Texas communities[b]	Front Range communities[c]
January	6.7	5.3
February	6.8	5.6
March	6.9	5.6
April	7.9	6.3
May	8.5	6.4
June	9.5	9.8
July	11.2	14.8
August	11.2	16.8
September	9.9	11.4
October	7.5	7.2
November	6.8	5.6
December	6.8	5.2

[a] Columns may not total 100 because of rounding

[b] From Griffin (1990), based on data from 221 communities. Figures are for 1981-85. Average annual use was 629 l/c/d.

[c] Weighted (by population) average 1995 deliveries of the following Colorado cities: Fort Collins, Loveland, Longmont, Greeley/Evans, Pueblo, and Denver. Average annual use in 1995 was 745 l/c/d.

Industrial Water Use

Industrial water use in the U.S. is dominated by use in thermoelectric power generation (table 2.1). For the country as a whole, thermoelectric power generation accounts for 40 percent of total withdrawals, with other industrial uses accounting for an additional 9 percent. However, the relative importance of industry is considerably less in the West, where thermoelectric power generation and other industrial uses account for 7 and 3 percent of total withdrawals, respectively. In some states industry uses even less water. In Colorado, for example, withdrawals for residential use alone are three times as large as withdrawals for all industrial uses (table 2.1).

The estimates of elasticity for industrial water use summarized in table 2.8 appear in aggregate to be larger than those for residential water (table 2.4). As with commercial uses, elasticity of demand for industrial uses varies among alternative types of industries (Renzetti 1992b, 1993, De Rooy 1974). Thus, the elasticity of demand in any urban area would be sensitive to the particular industries located there.

The cost of water is a very small part of total production costs in most industries (Bower 1966, Gibbons 1986), suggesting that industries might be able to pay considerable sums for water. However, because most industrial water users can recycle water in their production processes, they are unwilling to pay more for additional water withdrawals than the cost of installing or improving a recycling capability. Thus, the value of marginal withdrawals is approximated by the marginal cost of recycling. Gibbons (1986), citing Young and Gray (1972) and Russell (1970), reported marginal cost estimates of $9 to $20/acre-ft for cooling water and $91 to $134/acre-ft for industrial process water.

With complete recycling, withdrawal equals consumptive use. However, as seen in table 2.1, the industrial sector in general is far from achieving complete recycling. Thus, the value of withdrawals should approximate the marginal cost of recycling for a substantial proportion of the total industrial withdrawal. However, as withdrawal amount approaches consumptive use amount, the value of industrial water use must increase substantially. When this occurs, the demand curve for industrial water can be expected to rise sharply as quantity demanded approaches consumptive use. This suggests a nonlinear demand curve, convex to the origin, as could be depicted by a power or exponential function.

Unfortunately, specific estimates of an industrial water use demand curve are not generally available in the literature. If industrial water use is significant in a basin where water allocation is being analyzed using a model like AQUARIUS, a site-specific study may be necessary.

Hydropower

Electricity prices are so heavily regulated that they cannot be used as the basis for deriving the value of the water input at hydroelectric plants. As Young and Gray (1972) and Gibbons (1986) explained, the value of hydroelectric energy is therefore commonly estimated using the alternative cost technique, based on the reasoning that electricity not produced at hydroelectric dams will be produced at the next more expensive alternative. Use of the alternative cost valuation method is appropriate because the energy produced at a hydroelectric plant typically enters a large electric grid, composed of many hydro and thermal electric plants that supply numerous demand areas. The various plants in the grid are substitute suppliers, and the grid typically has some amount of excess capacity. If one supplier reduces production, other suppliers make up the difference. Typically, baseload power produced at a hydroelectric plant is replaced by a coal-fired plant and peaking power is replaced by a gas- or oil-fired turbine plant. In the Western U.S., gas-fired turbines are most common. More expensive oil-fired turbines also play an important role in the Eastern U.S.

Using the alternative cost method, the value of hydropower in dollars per acre-foot is estimated to be equal to 0.87 × feet of head × cost savings, where the cost savings is the cost per kilowatt hour (kWh) at thermal plants minus the cost per kilowatt hour at hydro plants. The 0.87 reflects the efficiency of converting the energy of falling water into electrical energy. The cost savings

may be determined for the short run or long run. In the short run, capital costs are fixed and the only costs of note are those for operation and maintenance (O&M). At thermal plants, O&M costs are dominated by fuel. Variable costs tend to be constant per unit of output, allowing simple average costs to be used regardless of the quantity of electricity. Gibbons (1986) reported that O&M costs at U.S. thermal plants averaged 18.52 and 44.01 mills/kWh at coal-fired and gas-fired plants, respectively, whereas the O&M costs at private hydroelectric plants averaged 1.52 mills/kWh (1980 dollars). The short-run values of hydropower averaged $0.0148 and $0.0370/acre-ft per foot of head for baseload and peaking power, respectively.

However, at about the time Gibbons (1986) published her report, the prices that electricity plants paid for fuel dropped and these lower prices have largely been maintained. Nationally, prices per kWh dropped to about 14 mills for coal and 23 mills for natural gas (Energy Information Administration 1996). These price reductions have had corresponding effects on total O&M costs at these plants. Further, prices in some areas of the West may be lower than these national averages. Accurate estimates of the value of hydropower require careful examination of costs at the affected thermal plants.

A long-run value would take into account the full costs of both types of plant and would include capital costs in addition to O&M costs. Estimating these costs is an involved process. Young and Gray (1972) provided one attempt to estimate the long-run value of hydropower. As Gibbons (1986) indicated, in an update of Young and Gray's work, the long-run cost savings per kilowatt hour may be smaller than the short-run cost savings. Most economic studies use short-run values. Currently in the West, an additional reason for using short-run values is the excess capacity in many areas.

Gibbons (1986) reminds us that using the alternative cost method described above ignores some costs, such as the environmental costs of thermal plants (e.g., air pollution) and hydro plants (e.g., inundation of a riparian area), and ignores the differences in the reliability of service between the two kinds of generating plants. However, similar costs of other water uses (e.g., nonpoint source pollution in irrigated agriculture) are also typically ignored when estimating water value. Such costs should be separately accounted for if economic efficiency is at issue.

Water Demand Curve in Hydroelectric Energy Production

Because per unit variable costs tend to be constant in electricity production, and the short-run cost savings is nearly constant per unit of output, demand for water at hydroelectric plants is aptly characterized by a horizontal demand curve. The point where this horizontal demand curve intersects the price axis depends on the relative costs of the hydroelectric plant and its least cost substitute. As described earlier, this depends on whether the hydroelectric plant produces peaking or baseload energy or some combination of the two. The annual and monthly demand curves would be identical except for their lengths, which indicate the capacities of the hydro plants for the respective time periods. This capacity constraint is best represented in AQUARIUS as a physical constraint.

AQUARIUS can also adopt a decreasing demand curve that reflects the changing return from hydroenergy during different periods of the day. This is classically represented by a step function that shows the change of the daily demand from a low during late night/early morning hours to a peak during normal business hours. See details in the chapter on benefit functions.

Riparian Recreation Water Use

Streamflow may have immediate effects on recreation, as well as lagged (future) effects. Immediate effects usually follow an inverted-U relation, with recreation quality improving with flow increases to a point but then decreasing with further flow increases (Brown et al. 1991). Consider the following examples. As flow increases from low levels, rafting quality improves because portages are avoided and rapids become more exciting but, if flow continues to increase, rafting quality deteriorates as rapids become unsafe or washed out (Shelby et al. 1992a). The scenic beauty of streamside views increases with flow as water begins to move freely and riffles appear but declines if flow level rises sufficiently to give the stream an over-full, flooded appearance (Brown and Daniel 1991). The quality of streamside fishing reaches an optimum at flows that moderately concentrate fish but still allow movement along the stream channel but declines at higher flows when the channel becomes flushed and wading becomes difficult or dangerous.

Lagged effects also occur for various recreation activities. For example, flows affect the ability of fish to propagate (Cheslak and Jacobson 1990), which eventually affects angler catch rates. Also, flows affect streamside vegetation, which over time affects the quality of streamside camping or picnicking (Shelby et al. 1992b).

Immediate and lagged effects on the quality of a recreation experience may translate into changes in individual willingness to pay for a recreation trip (quality effect) and in the number of trips taken (participation effect). Thus, the recreational value of a given flow level, *V(Q)*, has the following four effects:

$$V(Q) = \begin{pmatrix} \textit{immediate} \\ \textit{quality} \\ \textit{effect} \end{pmatrix} + \begin{pmatrix} \textit{immediate} \\ \textit{participation} \\ \textit{effect} \end{pmatrix} + \begin{pmatrix} \textit{lagged} \\ \textit{quality} \\ \textit{effect} \end{pmatrix} + \begin{pmatrix} \textit{lagged} \\ \textit{participation} \\ \textit{effect} \end{pmatrix} \tag{2.3}$$

Adapting the model of Duffield et al. (1992), the aggregate value of a change in flow Q in the current time period t is given by the partial total derivative of PV with respect to Q_t:

$$\frac{dPV}{dQ_t} = \left[P_t \left(\frac{\partial W_t}{\partial Q_t} + \frac{\partial W_t}{\partial P_t}\frac{\partial P_t}{\partial Q_t} \right) + W_t \frac{\partial P_t}{\partial Q_t} + P_{t+a} \left(\frac{\partial W_{t+a}}{\partial Q_t} + \frac{\partial W_{t+a}}{\partial P_{t+a}}\frac{\partial P_{t+a}}{\partial Q_t} \right) + W_{t+a}\frac{\partial P_{t+a}}{\partial Q_t} \right] (1+r)^{-a} \tag{2.4}$$

where P = participation quantity (e.g., in recreation visitor days), W = willingness to pay (WTP) per unit of P, a = a lag period, and r is a discount rate. The four terms within the brackets of equation (2.4) correspond to the four effects listed in equation (2.3). The two quality effects in equation (2.4) have two components: the direct effect of flow on WTP and a congestion effect. In all the partial derivatives of the effect of flow on WTP there is an implicit effect that the flow change has on the quality of the recreation experience; that quality change leads to the WTP change. Equation (2.4) may represent different kinds of recreation that are each affected by flow level.

Economists have been estimating the value of recreation for over 30 years using methods such as contingent valuation and the travel cost technique (U.S. Water Resources Council 1983). The first studies that attempted to determine the relation of streamflow to riparian recreation value were published in the early 1980s (table 2.10). Such studies have focused on specific activities such as fishing or white water boating. Different studies measured different effects of flow on recreation value. For example, Bishop et al. (1987) estimated the immediate quality effect, and Johnson and Adams (1988) estimated the lagged quality effect. Only one study (Walsh et al. 1980) estimated a congestion effect. In some cases, effects that were not estimated were assumed nil. For example, Bishop et al. ignored a congestion effect for boating because use was fully controlled by permit, and Daubert and Young (1981) ignored congestion because they found no difference in WTP between the least and most crowded days. In other cases, effects were simply ignored because of lack of interest or data, or because of the limits of the valuation method employed.

Two studies (Daubert and Young 1981, Hansen and Hallam 1991) achieved a value estimate that to some extent included both immediate and lagged effects. However, even in these cases interactions between immediate and lagged effects, such as the effect of current fishing pressure on future fish populations and catch rates, were ignored.

Most of the studies in table 2.10 calculated a recreational value of instream flow, usually in terms of acre-feet. Five of the studies estimated the change in marginal value of flow as flow changed, but others only reported average values or marginal values for a common flow level. All else equal, unit value of flow is expected to be greater the: 1) more recreation activities occur, 2) longer the stretch of river that receives recreation, 3) greater the number of trips received per activity per mile of river, 4) better the quality of the recreation experience, and 5) less is the river flow level. Bishop et al. (1987) reported an average value of about 50¢/acre-ft for fishing and boating recreation downstream of Glen Canyon Dam in the relatively high-volume Colorado River. Hansen and Hallam (1991) reported a wide variety of current marginal values across rivers, but values above $30/acre-ft were uncommon.

Shape of the Demand Curve for Instream Flow for Recreation

Daubert and Young (1981) and Duffield et al. (1992) presented demand functions that show the marginal value of instream flow dropping linearly with flow level from a maximum

Table 2.10 Studies of the economic value of streamflow for recreation.

				Measured effects of flow on value[b]					Economic values estimated		
				Immediate			Lagged		Trip	Fish caught	Flow volume[c]
Author	River	Method[a]	Activity	Qual.	Part.	Cong.	Qual.	Part.			
Bishop et al. (1987)	Colorado (AZ)	CV	Fishing; boating	X					X		X
Duffield et al. (1992)	Big Hole & Bitterroot (MT)	CV	Fishing; shoreline[d]	X	X				X		
			Fishing+shoreline	X	X						X
Daubert & Young (1981)	Poudre (CO)	CV	Fishing[e]	X			X		X		X
			Boating; shoreline	X							
Hansen & Hallam (1991)	Many in U.S.	CR	Fishing[e]		X			X			X
Harpman et al. (1993)	Taylor (CO)	CV	Fishing				X			X	
Johnson & Adams (1988)	John Day (OR)	CV	Fishing				X				X
Narayanan (1986)	Blacksmith (UT)	TC+CB	Fishing+shoreline		X				X		X
Walsh et al. (1980)	Seven in CO	CV	Fishing; boating		X	X			X		X
Ward (1987)	Chama (NM)	TC	Fishing+boating		X						X

[a] CV = contingent valuation method; TC = travel cost method; CR = cross-sectional analysis; CB = contingent behavior (i.e., contingent visitation).

[b] Qual. = quality effect; Part. = participation effect; Cong. = congestion effect.

[c] Dollars per acre-foot at selected flow levels. The relation usually applies across months of the season; Daubert and Young is an exception as they estimated separate relations for each month of the fishing season.

[d] Picnicking, camping, hiking, and relaxing.

[e] Combined current and lagged effects without accounting for interactions between them.

of \$25 and \$31/acre-ft, respectively, at 100 cfs, to minimums below zero at sufficiently high flows. Walsh et al. (1980) and Ward (1987) presented demand relations that show the marginal value of instream flow rising to a maximum of \$53 and \$41/acre-ft, respectively, at moderate flow levels and then falling below zero at sufficiently high flows. Similarly, Narayanan's (1986) demand function rises to a maximum value of \$1.32/acre-ft at moderate flows, then drops to near zero at very high flows. A rising marginal benefit curve at very low flows, as reported in the latter three studies, indicates that flows must reach some minimum level before additions to flow have the expected impact on the quality of recreation. The former two studies might also have found a demand curve that rises and then falls as flow increases if they had investigated very small flow levels (below 100 cfs). In any case, once the minimum flow level is reached, the evidence from these five studies is that the demand curve is downward sloping.

Demand Monthly Distribution

All but one of the above studies gathered data for the principal recreation season—usually from May to October—and essentially assumed that the seasonal relation applied to each month within the season. Daubert and Young (1981) estimated separate relations of flow to value for each separate month during the season; however, the amount of data per individual month was small. As a first approximation, the seasonal demand curve of aggregate WTP versus flow level could be applied to each individual month of the recreation season, based on the assumption that variation in recreation participation and quality across months is a function of changing flow level.

Nonuse Values

Nonmarket, or noncommodity, uses of water include not only recreation (discussed in the previous section) but also preservation. Whereas riparian recreation involves an actual trip to a river or stream, preservation of riparian areas and aquatic habitat does not. Nevertheless, preservation is valuable to many people, as evidenced by the millions of dollars annually donated to organizations such as the Nature Conservancy. Preservation is known by economists as a nonuse value.

Using contingent valuation, economists have attempted to estimate preservation value, focusing on categories of nonuse value called existence and bequest values (see Brown 1993 for a survey of nonuse value studies). A few of the nonuse value studies have focused on water flow. For example, Loomis (1987) studied the nonuse value of flows into Mono Lake, and Brown and Duffield (1995) studied the nonuse value of preserving flows in two Montana rivers. However, application of contingent valuation to estimate nonuse value remains controversial (Arrow et al. 1993, Portney 1994). No doubt nonuse values for preserving certain species or ecosystems are substantial; however, there is no consensus that current methods accurately measure such values. In the absence of nonuse values for preservation of streamflows, preservation concerns can be handled in AQUARIUS using physical constraints.

Aggregate Consumptive Water Demand

In reviewing various studies, we found that demands for different consumptive uses in Western irrigation areas relate to each other roughly as depicted in the upper graph of figure 2.4. This figure is from Kelso et al. (1973), who were conceptualizing water demand in Central Arizona. At one extreme, residential users are expected to demand relatively small quantities of water but be willing to pay relatively high amounts. At the other extreme, agricultural users are expected to demand relatively large quantities of water but be willing to pay relatively low amounts. Commercial and industrial users are expected to fall in between these two extremes.

Aggregate consumptive water demand in an area competing for the same water supply can be expressed as the horizontal sum of the individual demand curves, as in the lower graph of figure 2.4. Although AQUARIUS allows agriculture and M&I to be represented by their individual demand curves, the size of the water allocation problem may be reduced by combining these two categories of use into an aggregate demand curve.

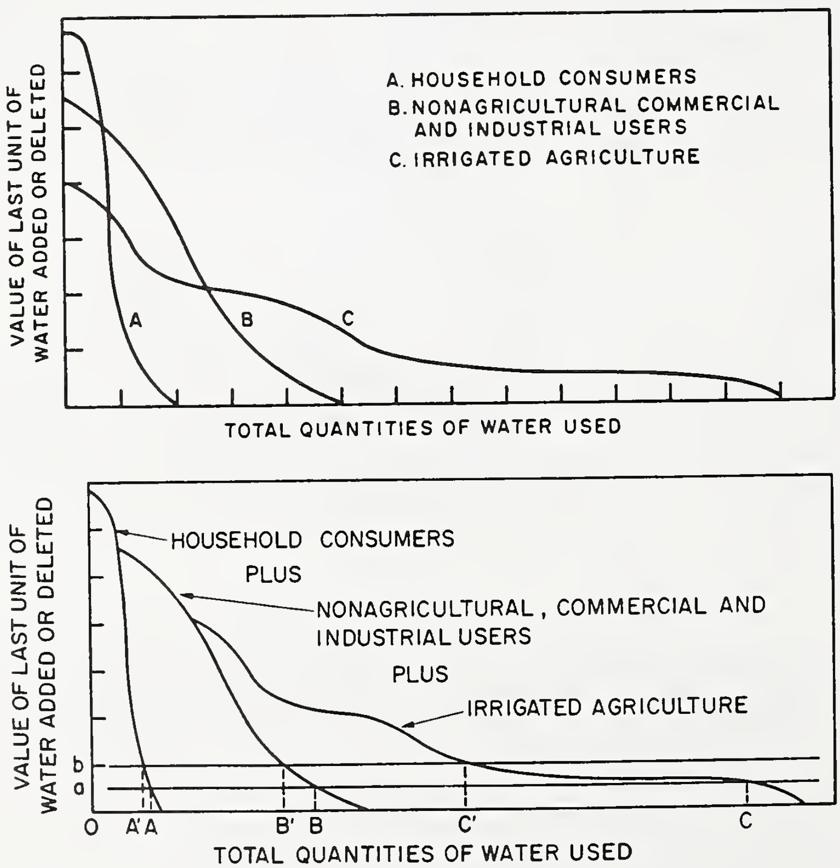


Figure 2.4 Individual demand curves (top); horizontal aggregation (bottom) (Kelso et al. 1973).

Final Remarks

Specifying demand curves is a difficult task. Although we know much about demand for different water uses, the precise location of a given demand curve is difficult to estimate (as evidence, observe the variety of elasticity estimates in tables 2.2, 2.4, 2.5, and 2.8). All attempts to plot demand curves must be regarded as approximate. In many situations, much extrapolation of demand in other locations, where demand studies have been done, is needed to proceed. Nevertheless, price-quantity relations do exist—though they may be difficult to precisely plot—and they do affect actual water allocation. Our understanding of efficient water allocation will be enhanced to the extent that we can model water allocation based on reasonably accurate demand functions. Even suboptimal solutions (based on approximate demand functions) are useful for understanding water allocation if they include all relevant water uses. With the growing demand for riparian recreation, this nontraditional water use can be equated with traditional uses in terms of how it is represented in the model and how an efficient allocation is reached.

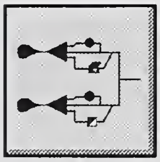
In this chapter we have provided information about the shape of typical demand functions for different water uses to facilitate the task of specifying demand curves when good site specific demand studies are lacking. Because any such specification is an approximation, sensitivity analysis is essential if the resulting optimization is sensitive to the demand specifications. AQUARIUS has been designed with such sensitivity analysis in mind.

Chapter 3

Modeling of System Components

This chapter presents functional objectives, basic principles of operation, and modeling assumptions for the various water-using sectors included in the model. This material provides background information for more detailed discussions in subsequent chapters about the development of benefit functions and the formulation of the general water allocation problem. The operational characteristics of the system components presented here address only those elements that are relevant for our modeling purposes. For detailed information concerning the management and operation of water systems, consult specialized publications like those from the U.S. Army Corps of Engineers (1986).

The River Basin System



Water resource engineering has benefited enormously from systems analysis, which has provided a framework for the conceptualization and analysis of river basin systems. Hall and Dracup (1970) define a system as:

". . . a set of elements which interact in a regular manner. Every system must have a well defined boundary or rule which specifies and distinguishes that which is in the system from the environment in which it exists. There will be inputs and outputs across the boundary which must also be defined."

This general definition aptly characterizes a water resource system. The elements interacting within the system, such as reservoirs, irrigation demand areas, and instream recreation activities, are called system components. System boundaries are defined by the flow network, which the analyst creates to represent the river basin to be modeled. The network specifies which components will be part of the system. Inputs, which consist of information necessary to simulate the operation of a system include, among others, natural flows, water demands for irrigation, instream flow requirements, and the price of energy. As explained latter, some of the inputs can be controlled by the analyst, but others are portrayed as uncontrollable inputs. Outputs are quantitative measures of the system performance that consist of many component objectives. Depending on the objectives, performance may be measured in terms of monetary values, amount of energy produced, biological indexes, or amount of water supplied to offstream demand areas.

A model is a conceptualization, and usually a simplification, of a real-world system. The degree of detail with which each component is characterized varies with the desired accuracy. For a model to reflect the real behavior of a system with accuracy, it must preserve all the essential operational characteristics of the river basin. For a generalized model, such as the one proposed here, this is a challenge.

Storage Reservoirs

The purpose of a storage reservoir (RES) is to transform the random and periodic nature of flows into a series of releases that more closely correspond with the seasonal water demands in a river basin. This objective is achieved by regulating the amount of stored water by passing flows through the reservoir outlet works and spillway to meet downstream water supply functions and to prevent flood damages. Flow regulation takes an uncontrolled flow, such as water flowing naturally in an undeveloped upstream basin, and turns it into a controlled flow, such as releases from a reservoir, to satisfy a particular demand. Figure 3.1 shows a sketch of a reservoir with all possible inflows and outflows.

XI = controlled inflows
d^u = upstream decision variable

UI = uncontrolled inflows
MR^u = upstream mandatory releases
NF^u = upstream natural flows
L^u = upstream reservoir spillage

XR = controlled releases
d = decision variable
MR = mandatory release

UR = uncontrolled releases
L = reservoir spillage
E = reservoir evaporation

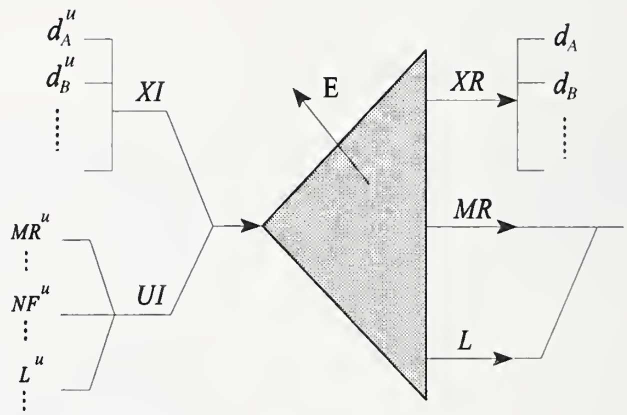


Figure 3.1 Variables used to describe storage dynamics in a reservoir.

A basic water balance equation that is used repeatedly in this study is:

$$dS(t)/dt = I(t) - O(t) \tag{3.1}$$

When equation (3.1) is used to describe the dynamics of storage in a reservoir, $dS(t)/dt$ is the rate of change of the reservoir storage with time, $I(t)$ is the rate of inflows into the reservoir, and $O(t)$ is the rate of outflows from the reservoir including controlled releases, spillage, and evaporation and seepage losses.

Analytical optimization techniques require that flows be classified as either controlled or uncontrolled flows. In figure 3.1, any upstream controlled flow, d^u, which reaches the reservoir under consideration, becomes a controlled inflow to the reservoir. If several d^u flows exist, they are all grouped under the term XI. Mandatory releases MR^u, spillage from upstream reservoirs L^u,

and natural flows NF^u are considered uncontrolled inflows to the reservoir and are grouped under the term UI. As with d^u, the inflow variables MR^u, L^u, and NF^u may be indicative of one or more sources of flow. Note that all inflows with the superscript $^{(u)}$ originated upstream from the reservoir under consideration.

Among reservoir outflows, we include the controlled releases d such as water released for a powerplant or an M&I demand area. As more than one controlled release is possible, we group them under the term XR. Uncontrolled reservoir releases, such as spills L and evaporation losses E, are under the term UR (not shown in figure 3.1) A third category of release, termed mandatory reservoir release MR, is used to satisfy any instream flow demand located immediately downstream from the reservoir. Only two water uses have the capability to demand instream releases from a reservoir. The first is an instream recreation area (IRA), where the released flow is considered a controlled reservoir release that becomes part of the XR term. The second is a fish habitat protection area (FHP) that can impose minimum and maximum instream flow requirements but does not have the capability to compete economically for water and, hence, is not a controlled release.

Using the notation indicated above and substituting the time index i for t to indicate discrete time intervals, the inflow to a reservoir during any time period i is expressed as a function of:

$$I(i) = f[XI_i + UI_i] = f[(d^u_{A_i}, d^u_{B_i}, \ldots) + (MR_i{}^u, NF_i{}^u, L_i{}^u)] \tag{3.2}$$

where the expression $(d^u_{A_i}, d^u_{B_i}, \ldots)$ is used to indicate the contribution of upstream control variables to the reservoir inflows. Similarly, total outflows from the reservoir are expressed by:

$$O(i) = f[XR_i + MR_i + UR_i] = f[(d_{A_i}, d_{B_i}, \ldots) + MR_i + UR_i] \tag{3.3}$$

where again the expression $(d_{A_i}, d_{B_i}, \ldots)$ groups all controlled reservoir releases and UR_i encompasses spills and evaporation losses.

The physical characteristics of a reservoir are defined by two basic functions: the elevation-storage curve, which defines the variation of the storage capacity in the reservoir with the water surface elevation; and the area-storage curve, which represents the area enclosed by the reservoir at different elevations. Both functional relations can often be approximated by the following power functions:

$$H = c_1 S^{d_1} \tag{3.4a}$$

$$A = c_2 S^{d_2} \tag{3.4b}$$

where S is storage volume in millions of cubic meters (Mcm), A is reservoir area in square kilometers (Km^2), H is the depth of water above a given datum in meters (m), and c and d are the parameters of the analytical models. Because of natural changes, such as sedimentation, occurring in the reservoir over time, the parameters may need to be recomputed after some years of operation. Other functional forms, such as a polynomial, can be used instead of (3.4a,b) if a good fit cannot be obtained with the power functions.

In addition to spills, two types of water losses to consider in a reservoir are evaporation and seepage. Both are functions of the level of storage in the reservoir. Evaporation is estimated by multiplying the surface area of the lake, computed using (3.4b), times the seasonal evaporation rate in millimeters. Evaporation rates can be estimated from pan evaporation data available for the region where the reservoir is located. Because precipitation over the lake is another source of inflow, the analyst should consider the net-evaporation rate, obtained as the difference between the pan-evaporation rate minus the expected precipitation depth during the season.

The consideration of seepage losses should be decided in a case-by-case basis. In general, assume that the amount of water lost by seepage increases with the level of storage in the reservoir because of the increase in the hydraulic gradient with additional storage. At present, the model does not account explicitly for seepage losses, although seepage losses can be imbedded into the evaporation losses as a first approximation.

AQUARIUS V96 considers a single storage pool for flow regulation. This volume S is defined between the maximum S_{max} and minimum S_{min} allowed operating storages, which are prescribed by the user (figure 3.2). In addition, the user must indicate the reservoir storage at the beginning of the operation and that required at the end of the optimization horizon, which are denoted as S^o and S^f, respectively. Reservoir elevation can be converted into reservoir storage by applying (3.4a). The single regulation pool in AQUARIUS contrasts with some other reservoir simulation models, such as HEC-5 (HEC 1989), where the storage in each reservoir is discretized into levels for operational control purposes. In fact, no more than a single operating pool is necessary when an optimization algorithm is used to determine the system operation, because the model finds the optimal strategies based on perfect foreknowledge of inflows and demands. Nevertheless, future versions of the model will probably define several operational levels in a reservoir, such as those included in HEC-5 (inactive, buffer, conservation, and flood control pools), to be able to simulate the historical system operation based on a set of reservoir operation priorities (reservoir operation rule curves).

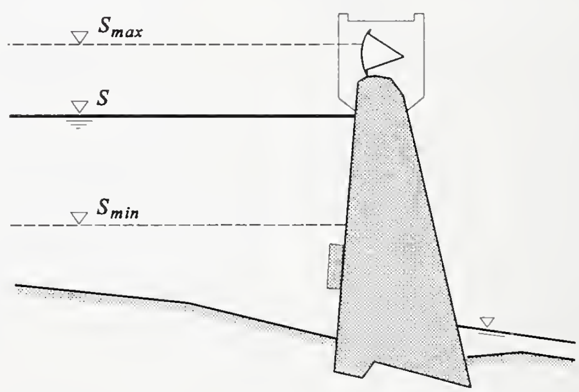


Figure 3.2 Reservoir regulation pool.

Practically all reservoirs have structures used to release floodwater that cannot be passed by other water passage facilities at the dam. Spillway crest gates are used to control the spillway discharge according to pre-established operation rules; ungated spillways are not subject to man-induced control. For the purpose of our modeling, we compute the total outflowing discharge, Q, of a reservoir through an ungated (uncontrolled) spillway by means of the following power function:

$$Q = C\, l\, H_1^{3/2} \tag{3.5}$$

where l is the net length of the spillway, H_1 is the total head-on crest including velocity head, and C denotes the coefficient of discharge of the overfall spillway. Knowing the water surface elevation in the reservoir, obtained from the elevation-storage function, and the elevation of the spillway crest (level of zero outflow), the model computes the volume spilled during the time interval of analysis.

For gate-controlled spillways (figure 3.3), outflow releases occur as orifice flow. The controlled discharge Q is computed mathematically by (3.6a), where H_1 indicates the head on the upper edge of the gate and H_2 is the head on the lower edge; both refer to the level of zero outflow. Again, C denotes the discharge coefficient, but for the case of a gate-controlled spillway, C is a function of the gate opening, as indicated by (3.6b). The coefficient of orifice discharge for various ratios of gate opening can be expressed in an analytical or tabular format and provided to the model by the user (not included in the present version of the model).

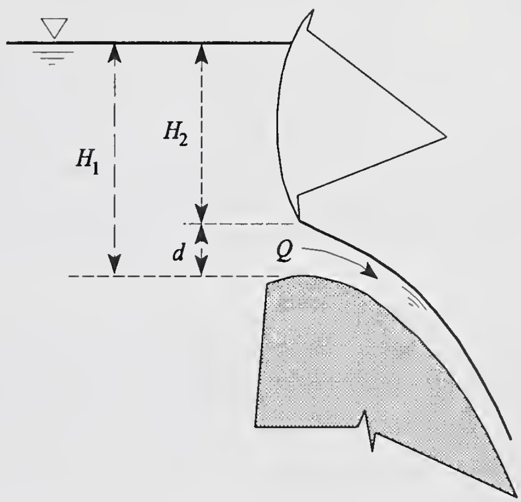


Figure 3.3 Section of a gate-controlled spillway.

$$Q = \frac{2}{3}\sqrt{2g}\; l\; C\,(H_1^{2/3} - H_2^{2/3}) \tag{3.6a}$$

$$C = f(d / H_1) \tag{3.6b}$$

The capacity of the spillway to pass a flood wave during a given time interval depends on the water surface elevation in the reservoir forebay at the beginning of the interval and on the length of the simulation time step. Iterative computational procedures are needed to estimate the gate-controlled spillway discharge coefficient C and to compute the residual reservoir storage volume

at the end of the simulation time step. Detailed simulation of the spillway operation becomes more relevant for small-time simulation steps (daily or hourly). For monthly or weekly time steps (depending on the size of the reservoir and inflow conditions), detailed operation of the spillway is usually omitted. For coarse time steps, the model passes any excess flood water downstream of the reservoir as a block, without attempting any spillway operation procedure. Details of the spillway operation simulation for short-time intervals will be provided in future versions of the model.

Hydropower Systems

Operation of an electric power system involves considering three basic components or subsystems, the: 1) energy sources (thermal, hydro, nuclear); 2) electric network components (generators, transformers, transmission lines); and 3) energy demand or load, constituted by a large number of electrical devices connected by the customers to the system. Each one of these components is a highly specialized engineering field that may require detailed modeling efforts depending upon the problem under consideration (e.g., El-Hawary and Christensen 1979). Our model deals only with the energy source component the hydropower (HPW) within it. Future additions of the model may include thermal energy sources to satisfy the electrical demand, providing the opportunity for a more comprehensive economical electric power operation.

In hydropower systems, hydraulic turbines convert the water energy stored in reservoirs or other structures into kinetic energy, which in turn is converted into electric energy by the generators. Hydropower facilities are typically hydraulically linked to reservoirs, and it is convenient to model the power plant and the reservoir as a unit. AQUARIUS can represent the following types of hydroelectric installations:

- storage plants, which have considerable water storage capacity in the reservoir that feeds the powerplant;
- run-of-river plants, which have negligible storage capacity and use water as it becomes available; and
- pumped-storage plants, which pump water from a lower reservoir to a higher reservoir (using inexpensive energy), to be run through turbines during on-peak demand periods. Pumped storage plants are not implemented in Version 96.

Storage plants can be further classified as variable-head plants and fixed-head plants. A hydropower installation is treated as a variable-head powerplant when the net hydraulic head acting on the turbines changes with water surface elevation changes in the reservoir. Typical layouts of variable-head plants show the powerhouse as: an integral part of the dam; a separate unit constructed at the toe of the dam (figure 3.4, left); or built at a distance from the dam and connected to the reservoir by tunnel and penstock. A fixed-head powerplant operates under a nearly constant net hydraulic head and is connected to the reservoir by a canal or some other type of open-channel structure (figure 3.4, right). This is also called a canal powerplant.

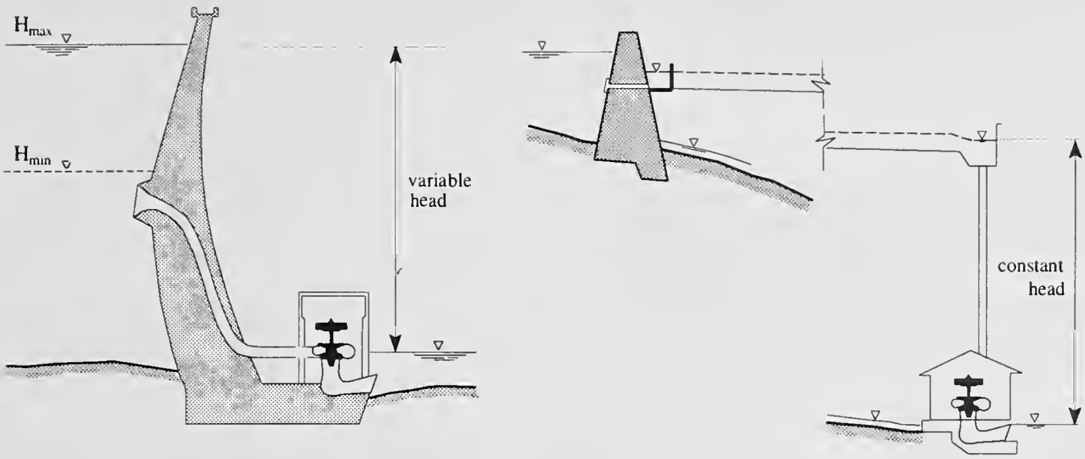


Figure 3.4 Layouts of storage hydro plants. Left = variable-head (H_{max} = maximum head; H_{min} = minimum head); right = fixed-head.

Flow regulation at the reservoir requires a decision to control the amount of flow being allocated from the reservoir to the storage powerplant. The model recognizes any link transporting water from a reservoir to a powerplant as a decision link, and creates the corresponding set of control variables.

The second basic type of power stations, run-of-river plants, are located in or alongside the main water course. Although several different layouts of run-of-river plants can be encountered, they all present: a diversion dam across the river with its intake; a canal that conveys the water into the forebay of the powerplant while gaining hydraulic head; the power-house; and another canal that routes the flow back to the river. The energy head acting on the hydraulic machinery is typically assumed fixed. This type of installation has insignificant impoundment at the powerplant. Therefore, water that cannot be passed through the hydro turbine runners because of a turbine shut down is spilled.

It is assumed that because flow going into a run-of-river plant is unregulated, the generated electric power is not a controllable variable. However, when the same run-of-river plant is analyzed within the context of a multi-user water system, it is necessary to make a decision as to how much water should be diverted into the powerplant and how much should remain in the river to comply with some instream use. As an example, figure 3.5 shows a fishery located in a river reach between the points of diversion and return of flows from a powerplant. The water allocation conflict between the two uses arises because of the need to limit the amount of flow that can be diverted into the powerplant in order to protect the fishery.

AQUARIUS considers any diversion node as a location in the flow network where a water allocation decision should be made. The diversion link branching out of the node becomes a

decision link when formulating the water allocation problem. In other words, even for run-of-river plants, the model converts the diversion link conveying water to the powerplant into a decision link and creates the corresponding set of decision variables.

The electric energy output of a hydropower plant is primarily a function of two variables: 1) the hydraulic head and 2) the rate of water passing through the turbines. The rate P, in kilowatts (kW), at which electrical energy can be generated is approximated by:

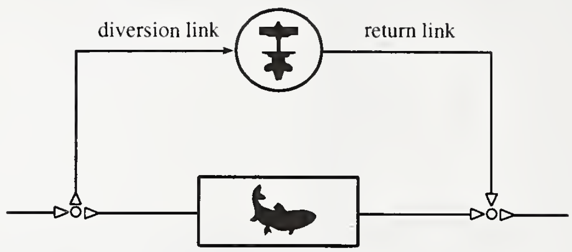


Figure 3.5 Power diversion in conflict with instream use.

$$P[\mathrm{kW}] = 9.8\, \eta_t\, \eta_g\, Q\, H_n \tag{3.7}$$

where Q is the rate at which water is discharged by the powerplant in cubic meters per second (m^3/s), H_n is the net (or effective) hydraulic head on the turbines in meters, η_t is the turbine efficiency, and η_g is the generator efficiency. Both efficiencies can be grouped into an overall efficiency η, which depends on discharge and effective head. The net head H_n in (3.7) is equal to the gross head minus energy losses before entrance to the turbine and outlet losses. The gross head of a powerplant is defined as the difference between forebay elevation and tailwater elevation.

Customarily, the capability of a powerplant to produce energy is expressed by the energy rate function (*erf*), which calculates the amount of energy in kilowatt hours generated by the plant per unit volume of water released through its turbines in cubic meters during a unit period of time (one hour):

$$erf[\mathrm{kWh}/m^3] = (1/367)\ \eta\ H_n \tag{3.8}$$

In its most general expression, the energy rate function (3.8) depends on the plant efficiency and tailwater conditions, which in turn are a function of storage in the reservoir and discharge. This is shown qualitatively in figure 3.6, which shows a smooth and practically linear variation of the *erf* with H_n within the normal range of operation of the powerplant. Complete modeling of the *erf* also requires knowledge of: 1) the tailrace elevation versus discharge function; and 2) a function that estimates losses of the energy grade line versus discharge.

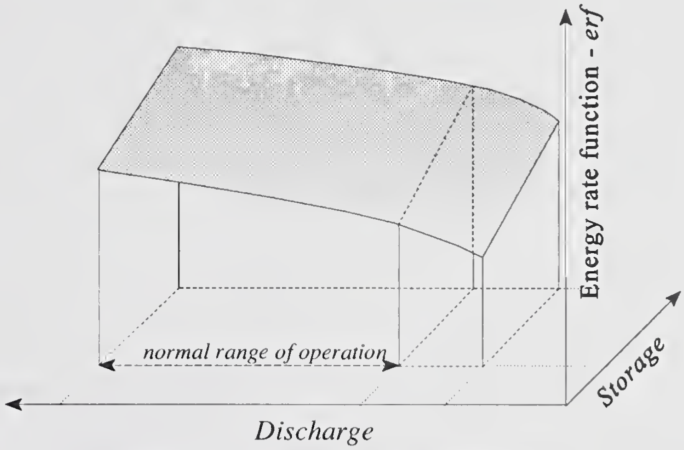
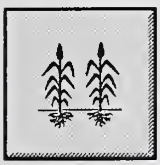


Figure 3.6 Energy rate function surface.

As presented later, the modeling of power production requires a continuous and differentiable form of the *erf*. As a first approximation, we can use the simple analytical expression given in (3.8), where the effect of discharge is implicitly considered within the variation of H_n and η. If necessary, a family of curves parametric on the discharge Q can be used to represent the surface in figure 3.6. When backwater effect from downstream reservoirs affects the tailrace elevation, special modeling considerations should be adopted, which are not in this version.

Agricultural Water Use



The lack of precipitation in many arid and semi-arid regions precludes crop production unless moisture is supplemented with irrigation (IRR) water. Historically, the need to ameliorate these moisture deficiencies has led to development of agricultural water supply projects, which are the largest consumptive users of water in a river basin. In the Western United States, such projects store runoff that occurs during the spring snow melt and releases it during the drier period of the growing season.

Large irrigation developments are composed of a complex array of water related structures for flow regulation, diversion, conveyance, distribution, and application. Once the irrigation water is applied to a field, additional structures may be required for capturing and transporting return flows back to the main water course. AQUARIUS deals with the first and last stages of the

agricultural water use system: 1) the supply of water to the demand area and 2) the physical link to return the nonconsumptive portion of that supply to the system. The model does not analyze the intricacies of the distribution and application subsystems.

Water for irrigation may be diverted directly (i.e., without prior storage) from a river to a farm if natural streamflow is sufficient. However, low flows in a river may severely limit the capacity of a natural system to reliably supply water, particularly for large irrigation developments. By creating a storage capacity between the water source and the demand area, the reliability of the water source increases dramatically. The net increase in dependable water supply, the firm yield, is a direct function of reservoir storage. Typically, a storage capacity that supplies irrigation water holds a portion of the incoming water in storage during the high flow season until the natural flow recedes to the point where it is no longer sufficient to meet the downstream irrigation demand. Releases of stored water from the reservoir are then used to augment flows in the river to meet the demand. Often the need for augmented flows persists until the end of the growing season. Even during the winter months, small amounts of water may still be allocated to the demand area for domestic use, stock water, or other purposes.

AQUARIUS can simulate the following possible schemes for providing water supply to an agricultural demand area:

- direct water diversion from existing flows in the river to an irrigation project;
- water released from a storage reservoir into a river and then withdrawn at a downstream diversion dam; and
- water released by gravity from a storage reservoir directly into a canal or conduit, rather than into the riverbed, and conveyed to the demand area. This also covers water pumped directly from the reservoir for irrigation.

Whether irrigation water is diverted from reservoirs, diversion dams or directly from natural river channels, the model individualizes the decision links in the network that are necessary to convey water to the demand areas and automatically creates the necessary sets of decision variables to control the supply.

The amount of water to supply to an agricultural area (water duty) must be specified by the analyst as AQUARIUS does not include an agricultural consumptive use submodel to estimate water demand for a given crop and condition. Water duties in the Unites States may range from about 3,000 to as much as 20,000 m^3/ha (1 to 7 acre-ft per acre). Losses in conveyance structures from the point of diversion to the point of application should be estimated by the analyst and added to the water duty. The return flow of water from irrigated lands may be collected in drainage channels where it flows back into a natural river channel at a known location. Alternatively, return flows may percolate into the regional aquifer to emerge as base flow at some location downstream in the river (distributed inflow). This return flow augments the prevailing river flow and, depending primarily on water-quality parameters, the return flow may be reused downstream by other users. As reported in the literature, the amount of return flow averages about 50 percent of the water diverted for irrigation purposes, varying from about 20 to 70 percent.

Return flows in the model are computed as a fixed percent of the flow allocated for the period, a percent (provided by the user) that is assumed constant for all seasons of the year. Version 96 of the model does not consider delays in the routing of flow return from the agricultural area to the site of recapture by the flow network. That is, the return flow is made fully available downstream during the same time interval of simulation. A more realistic conceptualization of return flow routing, necessary when considering return flows that move as subsurface flow and short simulation time intervals, will be included in a future version of the model. Whether return flows are transported back to the main watercourse using a conveyance structure or as groundwater accretion, the location in the flow network where return flows are regained should be indicated by the user during the process of creating the flow network. The return link is connected to the natural channel using a junction node (left- or right-bank). If no link is provided for return flows, applicable when representing a transbasin diversion, the total amount of water diverted is assumed consumed in the demand area.

Once the area under cultivation is defined, the maximum and minimum amounts of irrigation water required for consumptive use is fixed. Early estimates of the amount of agricultural water available for the growing season are used to predetermine the planting area. Then, the amount of water required to meet the demand for growing the crops for the entire season is also defined. Water should be supplied period after period during the whole growing season to cover the expected deficiencies in consumptive water use. Failure to supply the demanded water at any period may cause the complete loss of the area under cultivation, with corresponding economic loss. Figure 3.7 shows a typical pattern of water requirements for agricultural water in the Rocky Mountain region of the Western U.S., with the maximum demand occurring at the end of the summer. In contrast to the well defined growing season shown in figure 3.7, other agricultural areas have a year-round growing season. AQUARIUS allows the user to prescribe the total amount of water to be delivered to an irrigation area during the entire growing season, the seasonal pattern of the deliveries, and even the maximum and minimum volumes at each time interval.

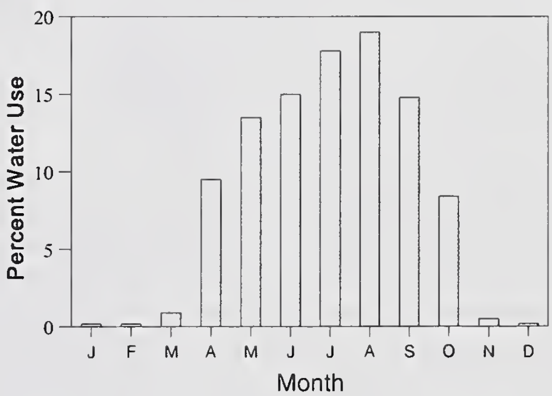


Figure 3.7 Seasonality of agricultural water use.

At present, surface water is the only source of irrigation water available in the model. In the future, we plan to incorporate surface and subsurface water to satisfy the demand of the agricultural sector. It is expected that as the marginal cost of acquiring surface water for satisfying irrigation demand increases, ground water may become an economical alternative.

Municipal and Industrial Water Use

Historically, urban water has been supplied separately from other uses, although municipal water supply intakes are sometimes provided in dams built for other purposes. For example, in some locations municipal water is conducted in combination with agricultural water where both use the same installations. A municipality water demand can be modeled as a single user using the system component described here, or it can be integrated with some other demand, using a combined demand curve. In this section, we concentrate on the basic operational characteristics and modeling of water supply for household, industrial, commercial, public and other activities, usually termed M&I. The model simulates the water demand in the M&I zone as a single block, transporting the allocated water from the flow regulation subsystem to the boundary of the M&I area. The model makes no consideration of the distribution subsystem inside the demand area.

Practically all river basin systems supply water for municipal and industrial use. Supply of M&I water in a highly reliable manner is usually given a high priority. As a whole, M&I water demand is relatively small compared with irrigation demand. The normal range of demand from an urban area is from 250 to 400 lpcd (liters per capita per day) (70 to 100 gallons per capita per day), although this varies considerably with the climate and economic conditions of the demand area. In arid and semi-arid regions with an abundant and low-cost water supply, the average annual demand can be up to twice those values. The amount of water supplied for municipal and industrial uses is fairly uniform through the year, except in areas with substantial lawn sprinkling. Although indoor use remains practically constant over the year, the sprinkling of open areas contributes to the seasonality of urban demand. Figure 3.8 shows the typical monthly distribution of water use for a city in a semi-arid region in the U.S. The almost uniform distribution in figure 3.8 contrasts with the highly seasonal demand in figure 3.7 for irrigation water.

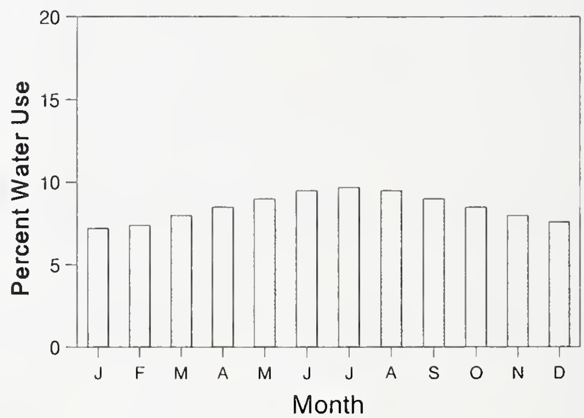


Figure 3.8 Seasonality of municipal and industrial use.

Similar to irrigation water, M&I water may be withdrawn directly from the stream using open channel flow diversions or from reservoirs that release water from storage into canals or pipes. The three schemes of water supply listed previously for agricultural water can also be simulated for M&I use by AQUARIUS. Whether M&I water is diverted from reservoirs, diversion dams or directly from natural river channels, the model detects decision links in the network that convey water into M&I demand areas and automatically creates the necessary sets of decision variables to control the supply.

Return flows from urban areas are collected by a network of sewers and transported to treatment plants before being finally restored to the river channel. It is generally reported that between 60 and 90 percent of the total water supplied to an M&I area is returned as waste water, with small changes in water quality after treatment. The return flow augments the prevailing river flow and, depending on water-quality parameters, it may be reused downstream by some other water users. Return flows in the model are computed as a percent (entered by the user) of the total allocated inflow; this percentage is held constant for all seasons of the year. Losses in conveyance structures from the point of diversion to the point of use should be estimated by the analyst and added to the water demand. As indicated earlier for irrigation return flows, the present version of the model cannot delay flows returning from the demand area to some later time step, although for M&I use this is less critical. While building the flow network, the user should indicate the junction site where M&I return flows are routed back to the main watercourse (using a left- or right-bank junction node). If no link for return flows is provided, the model assumes that the total amount of diverted water is consumed in the demand area.

Instream Recreation Water Use

The importance of instream water-based recreation (IRA), such as fishing, boating, swimming, and picnicking at the water's edge, has recently increased dramatically. Throughout earlier decades, when many of the reservoirs and diversion works in the U.S. were constructed, recreation was an incidental use of rivers, streams, lakes and reservoirs. After passage of the Flood Control Act of 1936, which required benefit-cost analysis of federal water development projects, recreation was often given formal recognition. Although the expected recreation at a new reservoir was counted as a benefit, the reduction in recreation quantity or quality in a dewatered channel downstream was ignored.

As water development continued to alter instream recreational opportunities and as incomes and leisure time grew, stream-based recreation received enhanced recognition. The federal government began to require more formal consideration of recreation in analyses of water developments. Consider the procedures for evaluation of water and related land resources promulgated by the Water Resources Council during the 1960s and 1970s, passage of the National Environmental Policy Act of 1969, the 1965 and 1972 amendments to the Clean Water Act requiring states to specify designated uses of its water bodies, and the 1986 amendments to the Federal Water Power Act requiring that the Federal Energy Regulatory Commission give equal consideration to conservation in the licensing of hydroelectric projects. Thus, reservoir management has moved from ignoring instream recreation to recognizing the impact of streamflows for recreation and esthetics purposes.

Because the value of stream-based recreation is related to the level of instream flow, quantification of the relation of instream flow to the quality of recreation for activities, such as rafting and kayaking, is becoming an important consideration in operating reservoirs and scheduling releases on many rivers (see Brown et al. 1991).

The attention given to instream flows goes beyond the demand for stream-based recreation. Colby (1990) comments that additional benefits are generated by the maintenance of instream flows, benefits related to the knowledge that the riparian ecosystem is preserved (nonuser values) and a stream's waste assimilation capacity (water quality benefits). Such benefits are often included with recreation benefits as conservation or environmental benefits. Because such inclusion is confusing, we, following the recommendation of the National Research Council (1992), separated water-based recreation from water use to protect ecosystem effects, fish and wildlife management, and habitat preservation, which is discussed later. The water use discussed in this section is the more commercially oriented water-related recreational activities, specifically, those occurring on natural waterways, excluding reservoirs and lakes.

A complex example of today's increasing demand for a host of water uses including high-quality water-based recreation is found in the Tennessee Valley. The growing number of requests from the local community prompted the Tennessee Valley Authority (TVA) to re-examine the operation of its large network of river-reservoir units, many of which were originally single-purpose power projects. The resulting study (TVA 1990) is a major effort to find a socially acceptable balance between the traditional uses of water in the TVA system, such as hydroelectric power production, navigation and flood control, and nontraditional uses such as recreation, water quality, and residential development.

A key decision variable in balancing competing water uses is the timing of reservoir drawdown (figure 3.9). For lake-based recreation and tourism, lakeside residential development, and some environmental issues, the longer high lake levels are maintained the better. However, delayed drawdown could cause water spills at the dams creating a risk of substantial hydropower losses, lack of flexibility in the operation of the power system, and increased pollution introduced by alternative energy sources.

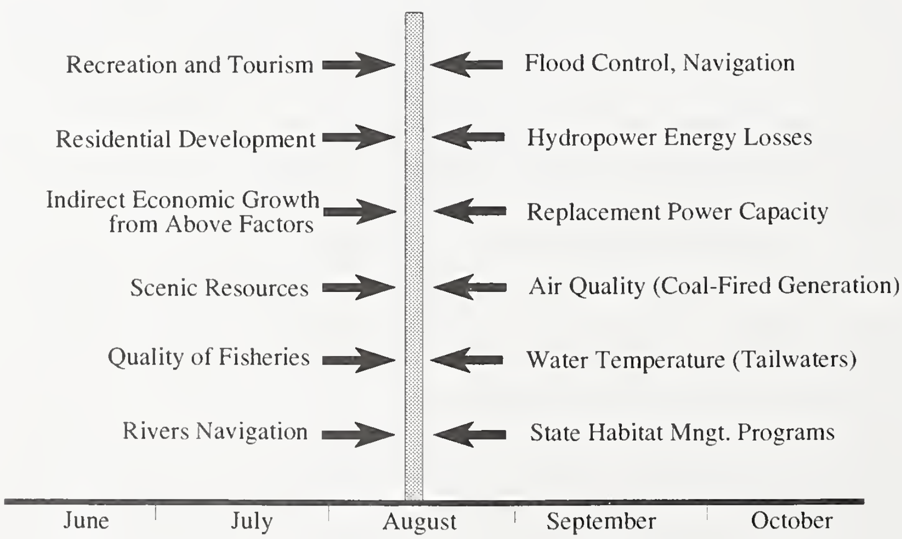


Figure 3.9 Forces acting on lake drawdown (TVA 1990).

Recreational use of streams has increased remarkably where whitewater floating is possible. For example, on the Cache la Poudre River of Northern Colorado, the number of people participating in commercial raft trips grew from 125 in 1984 to over 30,000 in 1995. During the same time, private river use by kayakers, rafters, and canoeists increased from a few hundred to about 9,000 trips per year. Similar increases have occurred on many rivers throughout the United States.

In addition to whitewater boating, many rivers have a large number of anglers and participants in water-enhanced activities such as camping and sightseeing. All this activity has a significant impact on local economies, where recreationists purchase goods and services related to their water-based activities. More difficult to measure but equally important is the value of the recreation to the participants. The value of stream-based recreation can be estimated in economic terms commensurate with the values of more traditional water uses by allowing such values to be incorporated into the multipurpose modeling process. Because stream-based recreation is not sold in competitive markets like many other water projects, the recreation value estimation requires specialized approaches. Nevertheless, this should not discourage attention to instream economic benefits. Recent studies (e.g., Duffield et al. 1992) have provided strong evidence that instream benefits can sometimes exceed those generated by offstream uses, and that gains are achievable on some systems by reallocating water from consumptive to instream uses. Similarly, Colby (1990) argues that instream values are high enough to compete in the market for water rights with offstream uses when important recreation sites and wildlife species are involved.

Note that water recreation has marked seasonal characteristics, similar to agricultural water use. The number of visitors can vary from very low during cold winter months to the very high during the spring and summer. For recreation, seasonality is also at the daily level as weekday use is typically much lower than weekends use. This suggests the need to use very short-time intervals when simulating the operation of a river system that includes recreation activities (see Morel-Seytoux et al. 1995).

The use of water for instream recreation is not exempt from conflicts with other uses. For instance, maintaining higher instream flows during the weekends increases opportunities for activities such as kayaking and boating but conflicts with electric power objectives because electricity demand tends to be lower during the weekends. Extra weekend releases may reduce the capability of the powerplants to meet energy and power demand during weekdays because of lower reservoir levels. Helping solve conflicts of this nature is the focus of multipurpose water system analysis.

Fish Habitat Protection

Fish and wildlife habitat protection (FHP) has become an important instream flow concern. In general terms, maintaining habitat involves protecting the ecosystem integrity of a river reach from excessive human activities. More specifically, such preservation can involve geomorphological concerns such as maintaining

sufficient sediment transfer capability, botanical concerns such as sustaining riparian vegetation, and water quality and temperature concerns. A detailed discussion of these topics is beyond the scope of this work. Our purpose is to recognize their importance in the analysis of flow networks and to provide some basic modeling elements.

Endemic fish populations throughout the U.S. are protected under the Endangered Species Act of 1973, and several recovery programs for fish and wildlife have been established by the U.S. Fish and Wildlife Service. Flow-habitat models are used to support flow recommendations to protect endangered fish. These models define the necessary flow conditions to protect and enhance biological species in a river (Lamb 1995), ideally, by re-establishing the natural variability of the flow regime lost by human-induced changes.

The two most common structural measures causing disruptions in natural flow regimes are construction of reservoirs that regulate flow and depletion of instream flows by offstream withdrawals. Figure 3.10 illustrates the effect of flow regulation in a snow-melt dominated basin. The graph, two mean annual hydrographs for the Black Canyon of the Gunnison River in Colorado, shows flows before and after reservoirs were built upstream in the mid 1960s. Note the drastic change in the pattern of flows caused by the presence of the reservoirs. Regulated flows are significantly higher outside the snowmelt season, and the natural peak in runoff occurring during the spring and early summer has been totally eliminated.

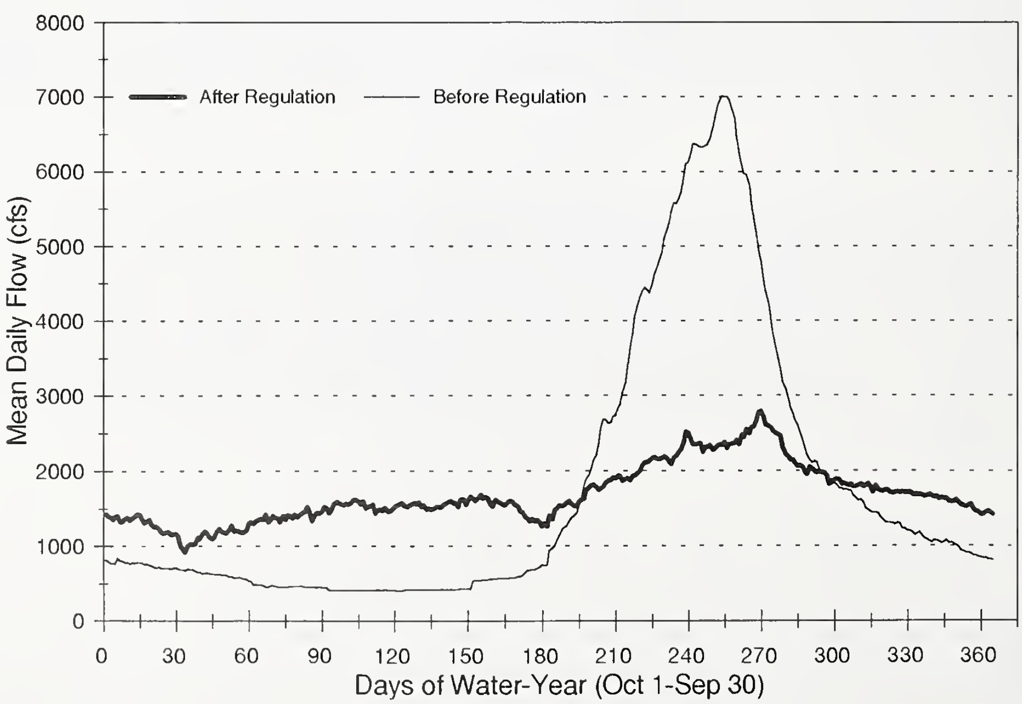


Figure 3.10 Annual hydrographs, pre- and post-regulation, for the Black Canyon of the Gunnison River.

The effects of flow regime alterations on habitat, and of habitat changes on species populations, are complex. Stream ecologists are, however, gaining a sufficient understanding of these relations, at least for some species, to recommend changes in water management to maintain habitat. Analysis is then needed on the feasibility of the changes, and on the effect of these changes on other water uses. Sometimes such analyses can be accomplished at a monthly time step, but other analyses require shorter time steps. For instance, the operation of a peaking hydropower facility can result in releases from zero (during off-peak hours) to maximum plant capacity (during on-peak hours). Although average flow values for the day may be adequate based on biological requirements, the within-the-day flow oscillations would be unacceptable. In addition to water quantity considerations, water quality is also critical. The release of cold-water from a reservoir can be detrimental for tailwater fisheries, creating ecological discontinuities in a colony of microinvertebrates that sustain the food web (see Stanford 1994).

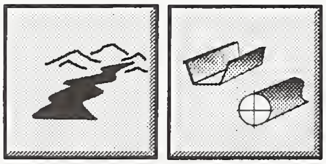
Although not in the current version of the model, we plan to incorporate elements of the Instream Flow Incremental Methodology (IFIM), developed by the U.S. Fish and Wildlife Service (see Bovee 1982), to quantify stream habitat availability. This additional analysis component will be a tool to measure the effect of the dynamics of flows on fish populations. IFIM has evolved into a comprehensive river model incorporating fish habitat, recreational space, and woody vegetation responses to alternative water management schemes (Stalnaker 1994). This is achieved by constructing time-series of habitat from the time-series of flows at selected points within a river system. The capability of AQUARIUS to estimate the quantity and quality of habitat at one or several locations in a basin, along with the other water uses, will enable scientists and decision makers to evaluate the adverse effects of planned or existing water developments and water transfers and identify opportunities for mitigation, restoration, or enhancement of affected instream and riparian resources.

Although economically significant fisheries are often identified, reliable estimation of nonuse and preservation values in economic terms is not possible. Therefore, AQUARIUS V96 does not assume that direct economic values for fish and wildlife habitat are available. This does not imply that this particular instream use of water has no economic value. Economic valuation research has confirmed that the American public attaches significant value to wildlife and fish preservation (Boyle and Bishop 1987), even though that value cannot be estimated with the same precision as other goods and services. Although the public's willingness to pay for kilowatt-hours of electricity generated, tons of corn produced, and whitewater kayaking opportunities can be quantified, quantification of the economic value for the preservation of endangered species and similar environmental concerns for direct incorporation in this model is insufficient.

What is proposed is to address the water allocation problem for noncommensurate values in a unique manner. For a river site where the level of instream flow has been predetermined (e.g., using the IFIM technique mentioned above), but where the demand function in economic terms is difficult or impossible to define, the analyst can experiment with fictitious demand curves until the required water allocation level for that instream use is satisfied. Indirectly, this approach indicates the societal willingness to pay for incremental increases of flow for difficult to value

water uses. In other words, this approach estimates the economic subsidy required to sustain the activity in open competition with the other directly valued uses in the basin.

Conveyance Structures



A river reach is the portion of a natural river system conveying flows between system components. The river reach, together with human-made conveyance structures, such as canals and pipelines, constitute the water system links of a river-flow network.

River reaches can experience different types of open channel flow, ranging from uniform to nonuniform flow and from steady to unsteady flow conditions, depending on whether the main flow characteristics are unchanged in space and time. The flow condition created in a natural channel when a reservoir discharges at varying flow rates creates unsteady flow conditions in the receiving channel requiring the consideration of time as an additional variable to describe flow in the river reach. This situation is illustrated in figure 3.11 in which releases from the upstream reservoir are modified by the river reach before entering the second reservoir.

The movement of a flood wave down a channel is associated with changes in timing and attenuation of the wave. The numerical analysis of this process is accomplished using some form of channel routing techniques. Any hydrologic routing technique is advised for this model. Hydrologic routing involves balancing inflow, outflow, and storage volume through use of the continuity equation (3.1). A storage-discharge relation is also required between outflow rate and storage in the system. There is a vast body of literature describing applications of hydrologic routing techniques (see Bedient and Huber 1988).

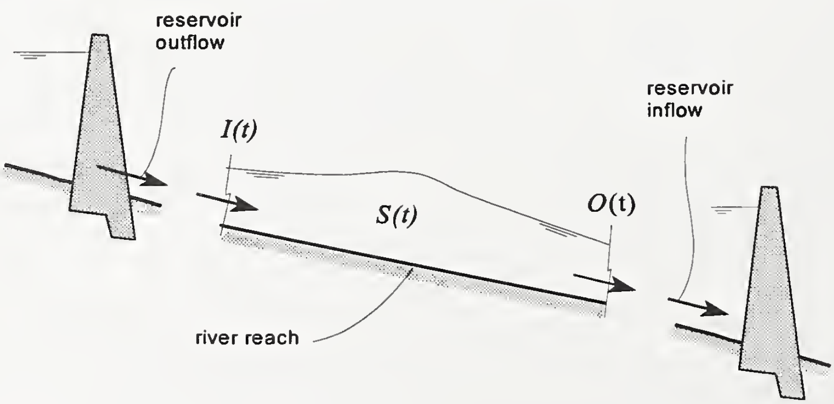


Figure 3.11 Flow routing in a river reach.

When equation (3.1) is used to describe the dynamics of storage in a river channel, $dS(t)/dt$ is the rate of change of storage within the reach, $I(t)$ is the inflow rate to the reach, and $O(t)$ is the outflow rate from the reach (figure 3.11). One of the most applied hydrologic routing procedures is the Muskingum Method, which assumes that outflow from a routing reach is a linear function of the sum of prism and wedge storage in the reach. Under this assumption, the Muskingum routing equation yields:

$$O_t = C_1 I_t + C_2 I_{t-1} + C_3 O_{t-1} \tag{3.9}$$

Equation (3.9) indicates that the channel output at a given time step is a linear combination of the inflow to the reach during the same time step and the inflow and outflow during the previous time step. The coefficients C_1, C_2, and C_3 can be computed once the storage time constant for the reach "K" and the weighting factor "x", the two Muskingum parameters, are known for the reach under consideration.

The time required for a flood wave to traverse the reach dictates the need for routing procedures. When the water allocation problem is simulated using long time intervals of analysis (e.g., weekly or monthly periods) it is assumed that all water entering a river reach is available at the downstream end of the reach within the same time interval of simulation. Hence, channel routing procedures are unnecessary. This can be verified by knowing the length of the river reach and a flow velocity for any representative flow rate. In contrast, when the time interval of simulation is short (e.g., 24 hours or less) streamflow routing is generally required to predict the temporal variations of a flood wave as it travels from the beginning to the end of the river reach.

Other Water Uses Not Included in the Model

The main goal of the first version of the model is to provide the general framework and basic elements for the analysis of river networks. As future opportunities permit, additional water uses will be incorporated into the model, allowing for a more comprehensive analysis of multipurpose water projects. The following water uses are candidates for future inclusion in the model:

Flood Control Area (FCA): Flooding, the overflow of water in land areas alongside a river or stream that would not normally be inundated, can cause severe economic and human losses. The operational objective of flood control is to minimize the frequency and the magnitude of flood damages by implementing a series of structural and nonstructural control measures, although the risk of damage by extreme events is rarely eliminated. Flood control is a common objective of reservoir management.

Reservoir Recreation Activities (RRA): In addition to the recreation activities described previously, other water-related recreational activities are practiced by the general public on and around reservoirs and lakes such as boating, fishing, water skiing,

camping, hiking and, again, fishing. Residential development is also a common shoreline use where the land is privately owned. The optimum recreational use of reservoirs requires that water levels be maintained practically constant and fairly high. Changes in water surface elevation caused by hydropower regulation can cause beach erosion affecting shoreline properties, make marinas unaccessible, and affect boat docks due to ice damage during the winter, among other problems.

Instream Water Quality (IWQ): Changes in the physical and chemical characteristics of instream flow are triggered by point and non-point pollution sources in the watershed. As surface flow levels are depleted, pollutants become concentrated and water quality standards (e.g., dissolved oxygen level) may be violated. Enhancing instream flows provides economic benefits by mitigating the treatment cost that would be incurred by discharges and by downstream water users to ensure compliance with federal and state standards (Young and Gray 1972).

Subsurface Water Supply (SSW): The current version of the model ignores ground water as a potential source. Considering the conjunctive use of surface and ground water will add a new dimension to the water supply component, allowing for water transfers between surface and ground water resources. In general, the cost of groundwater is higher than the cost of surface water due to pumping costs. However, where surface water is scarce, the cost of ground water may be less than it is for surface water, making augmentation of the supply using ground water sources a viable economic alternative. Note that where surface and ground water are hydraulically connected, the use of ground water can alter surface flows, with potential adverse effects on downstream third parties.

Channel Maintenance Flow (CMF): The volume and time distribution of water supplied to a river reach, among other factors, govern the physical processes in the river and hence, its morphology (i.e., channel-forming flows). Riparian plant communities are also significantly affected by the level and frequency of channel inundation.

Commercial River Navigation (CRN): Commercial navigation requires sufficient flow to allow ship passage. Instream flow needs for navigation may conflict with other water uses. For example, periodic fluctuations in reservoir releases from power peaking operations may disrupt navigation.

Chapter 4

Benefit Functions

This chapter presents the benefit functions associated with different water uses in a river basin. The following uses are considered: 1) hydropower with variable-energy head; 2) hydropower with fixed-energy head; 3) water for irrigation demand areas; 4) water for municipal and industrial demand areas; and 5) water for instream water recreation activities. The basic operational characteristics and modeling assumptions for the system components are in the previous chapter.

Variable-Head Powerplant

Hydropower generation is a function of the powerplant discharge, the hydraulic head under which the turbines operate, and the efficiency of the turbine-generator group. In turn, the effective head on the powerplant changes with elevation changes of the water surface in the reservoir forebay and tailrace. The energy rate function (*erf*), as defined in (3.8), can be simplified by assuming effective head to be a linear function of reservoir storage. In this formulation, storage serves as a surrogate for reservoir water surface elevation, and any effect of backwater on the powerplant tailrace level is ignored. The *erf* is expressed in megawatt-hours (MWh) per million cubic meters (Mcm) of flow through the turbines:

$$erf\,[\text{MWh/Mcm}] = \eta\,(a_1 + b_1 S) \tag{4.1}$$

where η encompasses all the mechanical and electrical efficiencies of the generation unit, and a_1 and b_1 are parameters of the linear model.

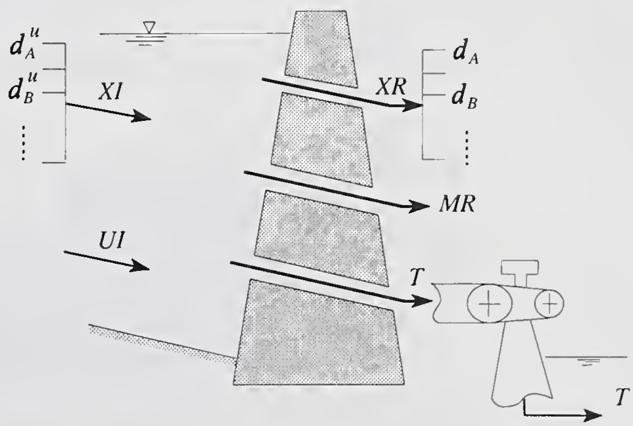


Figure 4.1 Inflows and outflows for a variable-head powerplant.

In this formulation, energy production during a given time interval depends on the average reservoir storage during that same time interval. Average storage $\bar{S}_i$ during time interval i can be estimated as the storage at the beginning of the period S_i^o, plus half of the inputs and outputs from the reservoir during the period. Using the notation established in

Chapter 3 and figure 4.1, the average storage during the time interval is written as:

$$\bar{S}_i = S_i^o + \frac{(UI_i + XI_i)}{2} - \frac{(T_i + XR_i + MR_i + UR_i)}{2} \tag{4.2}$$

where the powerplant release T has been segregated from the general group of controlled reservoir releases XR. To maintain unit consistency throughout the model formulation, flow variables denote water volume.

The price for energy sold by powerplants is usually stipulated by the power market to which the utility sells its energy. It is assumed that an additional powerplant does not significantly change the power pool, and that it has no effect on the energy price structure. This is generally true except for small, isolated electrical systems where an additional power utility can significantly affect energy supply and thus price.

For most electrical systems the price of energy depends on the time of day that the energy is delivered to the electrical grid. The highest prices are paid for energy delivered during the high demand on-peak hours. Additional amounts of energy, if available, are sold at lower unit prices during off-peak demand periods and during the night. This is the core of the classical hydro-scheduling problem. The three classes of energy are represented in figure 4.2 by a step function, showing the marginal price paid for energy (where a mill is one-tenth of a cent) as a function of the plant utilization factor.

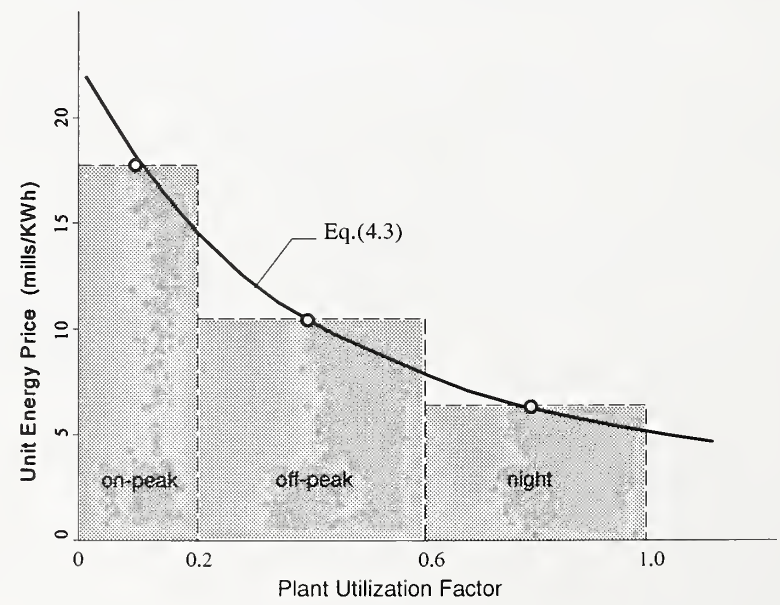


Figure 4.2 Demand function for hydro-energy.

The utilization factor indicates how the available water is used by the hydropower plant, within the constraint of powerplant capacity and characteristics. In this model, the utilization factor is the percent of plant operation time during the time interval of analysis (monthly, weekly, daily). In turn, the operation time can be replaced by the flow percent passing through the turbines, assuming that the generation units operate at a nominal flow rate. Note that the structure of prices may remain constant all year or change from season to season; summer prices for energy can be higher than winter prices.

The diminishing returns for hydropower in figure 4.2 can be analytically represented by the exponential function in (4.3), which provides adequate flexibility to represent downward sloping demand curves:

$$b\,[\$/\mathrm{MWh}] = \frac{P_S}{P}\, a_2 \exp(-T/b_2) \tag{4.3}$$

where T denotes the powerplant release (volume released during the time period), a_2 and b_2 are parameters of the exponential function, and P_S accounts for possible seasonal changes in energy prices with respect to the average annual price P.

The marginal price function for hydropower assumes that power releases are drawn first during on-peak hours, then during off-peak hours, and whatever volume of water is left will be used to generate night (dump) energy. Because the total number of on-peak, off-peak, and night hours of energy demanded by the electrical grid changes depending on the time interval of simulation (daily, weekly, or monthly) , the exponential demand curve in figure 4.2 must be built for the case under consideration and will yield different values for parameters a_2 and b_2.

Following Laufer and Morel-Seytoux (1979), integration of the product of the energy rate function (4.1) times the marginal price function (4.3) with respect to the powerplant release T_i, yields the hydropower return function that computes the benefit during a given time interval i:

$$B_i^{\mathrm{HPW\ VH}}(\eta, T_i, \bar{S}_i) = \int_0^{T_i} erf(\bar{S})\ b(z)\ dz \tag{4.4}$$

where z is a dummy variable of integration and all other terms have been defined earlier. Substituting the final expressions of $erf(\bar{S})$ and $b(T)$ into the integral above, the following expression is obtained:

$$B_i^{\mathrm{HPW\ VH}} = \int_0^{T_i} \eta \frac{p_i}{P} a_2 \exp(-z/b_2)\ [a_1 + \frac{b_1}{2}(2S_i^o + UI_i + XI_i - z - XR_i - MR_i - UR_i)]\ dz \tag{4.5}$$

By further expanding (4.5), (4.6) is obtained:

$$B_i^{\text{HPW VH}} = \eta \frac{p_i}{P} \frac{a_2 b_1}{2} (2a_1/b_1 + 2S_i^o + UI_i + XI_i - XR_i - MR_i - UR_i) \int_0^{T_i} \exp(-z/b_2)\, dz \\ - \eta \frac{p_i}{P} \frac{a_2 b_1}{2} \int_0^{T_i} z \exp(-z/b_2)\, dz \tag{4.6}$$

Carrying out the integration and regrouping some terms, the economic benefit from the operation of a single powerplant during time period i is obtained:

$$B_i^{\text{HPW VH}} = \eta \frac{p_i}{P} \frac{a_2 b_1 b_2}{2} \Big\{ [2a_1/b_1 + 2S_i^o + UI_i + XI_i - XR_i - MR_i - UR_i] [1 - \exp(-T_i/b_2)] \\ + [(T_i + b_2) \exp(-T_i/b_2) - b_2] \Big\} \tag{4.7}$$

Note that the storage S_i^o at the beginning of period i depends on the history of inflows and outflows from the reservoir up to that period, expressed as:

$$S_i^o = S^o + \sum_{k=1}^{i-1} (UI_k + XI_k) - \sum_{k=1}^{i-1} (T_k + XR_k + MR_k + UR_k) \tag{4.8}$$

where again the turbine release T has been separated from the term XR that represents other controlled reservoir releases. Substituting (4.8) into (4.7), the total benefit accruing from hydropower generation (with variable-head) in its expanded form is obtained:

$$B_i^{\text{HPW VH}}[\$] = \eta \frac{p_i}{P} \frac{a_2 b_1 b_2}{2} \Big\{ [2a_1/b_1 + 2S^o + 2\sum_{k=1}^{i-1} (UI_k + XI_k) - 2\sum_{k=1}^{i-1} (T_k + XR_k + MR_k + UR_k) \\ + (UI_i + XI_i - XR_i - MR_i - UR_i)] [1 - \exp(-T_i/b_2)] \\ + [(T_i + b_2) \exp(-T_i/b_2) - b_2] \Big\} \tag{4.9a}$$

where all terms have been defined earlier. The lengthy equation (4.9a) is a simple expression that computes the hydropower benefit as a function of the powerplant release, the history of inflows to and outflows from the reservoir that regulates flows for the powerplant, and a series of parameters that represent physical and economic data associated with the hydropower utility. This expanded form of the benefit function promotes the partial derivatives in Appendix A. Equation (4.9a) can be written in a more compact form by expressing it in terms of $\bar{S}_i$, the average reservoir storage during the interval i:

$$B_i^{\mathrm{HPW}^{VH}} [\$] = \eta \frac{p_i}{P} a_2 b_2 \left\{ [a_1 + b_1 \bar{S}_i - \frac{b_1 b_2}{2}] [1 - \exp(-T_i/b_2)] + \frac{b_1 T_i}{2} \right\} \tag{4.9b}$$

Fixed-Head Powerplant

The hydropower benefit function in (4.9) can be simplified for a run-of-river powerplant for which the energy rate function (4.1) reduces to a single value because of the fixed energy head acting on the hydraulic turbines, that is:

$$erf\,[\mathrm{MWh/Mcm}] = \eta\, a_1 \tag{4.10}$$

The marginal price of energy is again indicated as in (4.3), where T denotes the total volume of water passing through the turbines (figure 4.3).

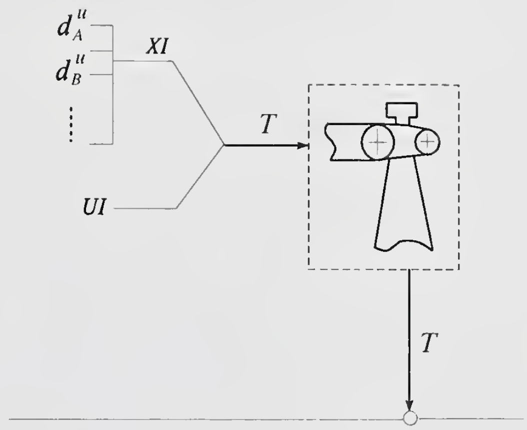


Figure 4.3 In/outflows for a fixed-head powerplant.

Again, the integral of the product of the energy rate times the demand function with respect to the powerplant release yields the hydropower benefit function. Using the final forms of *erf* and b in (4.10) and (4.3), the following expression is obtained:

$$B_i^{\mathrm{HPW}^{FH}} = \int_0^{T_i} \eta\, a_1 \frac{p_i}{P} a_2 \exp(-z/b_2)\, dz \tag{4.11}$$

where z is a dummy variable of integration. Carrying out the integration, the economic benefit from the operation of a fixed-head powerplant during time interval i is:

$$B_i^{\mathrm{HPW}^{FH}} [\$] = \eta \frac{p_i}{P} a_1 a_2 b_2 [1 - \exp(-T_i/b_2)] \tag{4.12}$$

In contrast to a variable-head powerplant where a single controlled reservoir release is allowed to enter the powerplant, the flow T into a fixed-head powerplant can be a function of several controlled and uncontrolled flow variables. The most general case is depicted in figure 4.3 and expressed mathematically as:

$$T_i = f[XI_i, UI_i] = f[(d_{A\,i}^u, d_{B\,i}^u, \dots), UI_i] \tag{4.13}$$

where XI is substituted by $d^u_{A_i}, d^u_{B_i}, \ldots$ to indicate possible multiple controlled flows entering the powerplant, and UI is all uncontrolled flows reaching the powerplant. Substituting (4.13) into (4.12), the expanded form of the fixed-head hydropower benefit function is:

$$B_i^{HPW^{FH}}[\$] = \eta \frac{p_i}{P} a_1\, a_2\, b_2 \left\{ 1 - \exp[(-c^u_A d^u_{A_i} - c^u_B d^u_{B_i} - \ldots - UI_i)/b_2] \right\} \tag{4.14}$$

where $c^u_A, c^u_B, \ldots$ are the coefficients of the upstream decision variables $d^u_A, d^u_B, \ldots$, respectively.

Irrigation Demand Area

As discussed in Chapter 2, the benefit from an irrigation demand area (IRR) is determined by the net increase in income from the cultivated irrigated land. Typical of agricultural areas, the IRR demand curve includes large amounts of low value water. In this model, the marginal value of a given unit of flow used for irrigation, expressed in dollars per million of cubic meters ($/Mcm), is assumed to follow an exponential decreasing function (figure 4.4):

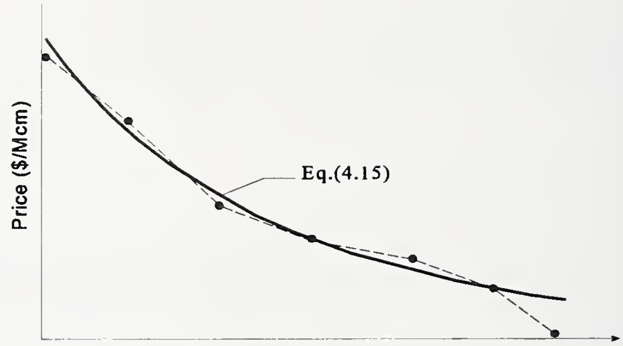


Figure 4.4 Annual irrigation demand.

$$b^{IRR}[\$/\text{Mcm}] = a_3 \exp(-A/b_3) \tag{4.15}$$

where A represents the total volume of water entering the IRR area as indicated in figure 4.5, and a_3 and b_3 are the parameters of the exponential function. In general, the decaying model in (4.15) provides good fittings of the marginal-benefit curves derived in practice, except for the lower portion of the curves. While the exponential function becomes asymptotic to the axis, actual demand curves will intercept the horizontal axis (see figure 2.2).

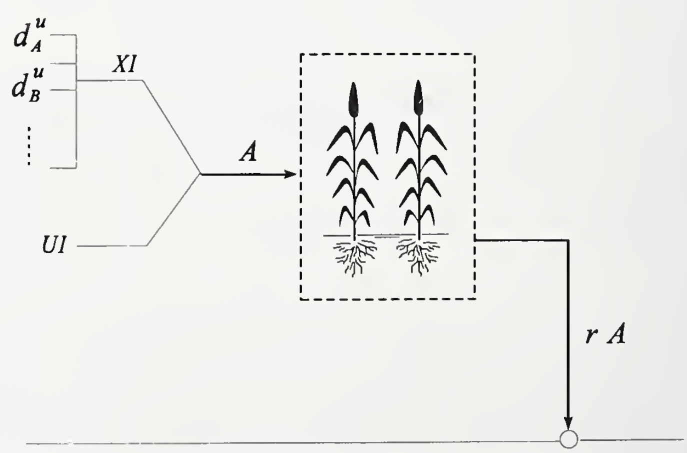


Figure 4.5 In/outflows to an irrigation demand area.

Part of the flow diverted into the agricultural zone is consumptively used, but a fraction of A returns to the stream via subsurface flow or drainage systems. Once the return flow reaches a stream, it is available for use downstream. This fraction of the incoming flow, indicated by r in figure 4.5 is the return flow

coefficient of the irrigation zone. The return flow coefficient is assumed constant for all seasons and can be highly variable from one irrigation area to another depending on irrigation practices and soil characteristics.

The integration of (4.15), which is the area under the demand curve, yields the total benefit from the IRR demand area (i.e., the total willingness to pay for irrigation water in the demand area):

$$B_i^{\mathrm{IRR}} = \int_0^{A_i} a_{3_i} \exp(-z/b_{3_i})\, dz \tag{4.16}$$

where z is a dummy variable of integration. Solving the integral in (4.16), the IRR benefit function for a single time interval i is obtained, where the parameters a_3 and b_3 can vary by time interval (i.e., seasonally):

$$B_i^{\mathrm{IRR}}\,[\$] = a_{3_i} b_{3_i} \,[1 - \exp(-A_i/b_{3_i})] \tag{4.17}$$

As depicted in figure 4.5, the total flow A entering the IRR area can be several controlled XI and uncontrolled UI inflows. Mathematically:

$$A_i = f[XI_i, UI_i] = f[(d^u_{A_i}, d^u_{B_i}, \ldots), UI_i)] \tag{4.18}$$

where the term XI is expressed by $d^u_{A_i}, d^u_{B_i}, \ldots$ to indicate possible multiple controlled flows entering the agriculture irrigation zone. Substituting (4.18) into (4.17), the expanded form of the benefit function is:

$$B_i^{\mathrm{IRR}}\,[\$] = a_{3_i} b_{3_i} \left\{1 - \exp[(-c^u_A d^u_{A_i} - c^u_B d^u_{B_i} - \ldots - UI_i)/b_{3_i}]\right\} \tag{4.19}$$

where $c^u_A, c^u_B, \ldots$ are the coefficients of the upstream decision variables $d^u_A, d^u_B, \ldots$ respectively.

Municipal and Industrial Demand Area

Empirical studies indicate that the quantity of water demanded by the municipal and industrial (M&I) sector is sensitive to price (Martin and Thomas 1986, Howe, 1982, Howe and Linaweaver 1967, Foster and Beattie 1979, Lyman, 1992) but not as sensitive as irrigation demand. Compared with agricultural, the M&I sector demands limited quantities of water but is willing to pay relatively higher prices; M&I demand tends to be relatively inelastic. The analytical form of the M&I demand curve available in the model also follows an exponential decaying model:

$$b^{M\&I}\ [\$/\text{MCM}] = a_4 \exp(-D/b_4) \tag{4.20}$$

where D represents the total volume entering the M&I area and a_4 and b_4 are parameters of the exponential function. In general, only a small portion of the flow D diverted into the demand zone is consumptively used. The unused portion, indicated by the return flow coefficient r in figure 4.6, becomes wastewater flow that, after treatment, is available to other users.

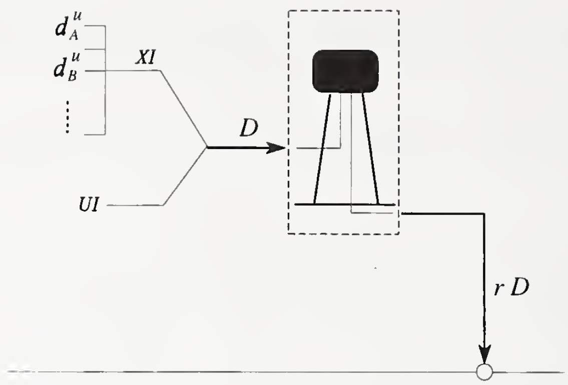


Figure 4.6 In/outflows to a municipal and industrial zone.

The integral of the demand curve (4.20) yields the total benefits stemming from the municipal and industrial water use:

$$B_i^{\text{M\&I}}[\$] = \int_0^{D_i} a_{4_i} \exp(-z/b_{4_i})\ dz \tag{4.21}$$

where z is a dummy variable of integration. Solving the integral, the M&I benefit function for a single time interval i is:

$$B_i^{\text{M\&I}}[\$] = a_{4_i} b_{4_i}\ [1 - \exp(-D_i/b_{4_i})] \tag{4.22}$$

Note that in (4.22) the parameters a_4 and b_4 can vary seasonally. As in the irrigation case, the amount of water D entering the M&I area can be several controlled and uncontrolled flows (figure 4.6). Mathematically, the M&I diversion is expressed as:

$$D_i = f[XI_i, UI_i] = f[(d_{A_i}^u, d_{B_i}^u, \ldots), UI_i] \tag{4.23}$$

where the controlled inflow XI is replaced by $d_{A_i}^u, d_{B_i}^u, \ldots$ to indicate possible linear combinations of several control variables contributing to flows to the M&I zone. Substituting (4.23) into (4.22), the expanded form of the M&I benefit function is:

$$B_i^{\text{M\&I}}[\$] = a_{4_i} b_{4_i} \left\{1 - \exp[(-c_A^u d_{A_i}^u - c_B^u d_{B_i}^u - \ldots - UI_i)/b_{4_i}]\right\} \tag{4.24}$$

where $c_A^u, c_B^u, \ldots$ are the coefficients of the upstream decision variables $d_A^u, d_B^u, \ldots$ respectively.

Instream Recreation Area

Because instream recreational (IRA) opportunities are not generally sold in a market, estimating the benefits from water use for recreation requires unique economic valuation approaches such as the travel cost method and the contingent valuation method (Freeman 1993). Brown et al. (1991) list many of the studies performed over the last 20 years that used these methods to focus on the recreational value of instream flow.

For example, Duffield et al. (1992) used a dichotomous choice, contingent valuation survey to interview recreationists along the "blue ribbon" trout fishery rivers in Montana. Interviews were conducted through the summer, and flow conditions were recorded for each interview day. The information allowed the investigators to develop a relationship between willingness to pay (WTP) for instream recreational participation and alternative flow conditions. According to Duffield et al. (1992), the marginal value of a given flow unit is the effect of instream flows on recreational experience quality, quantity of use, and lagged effects on recreation conditions. It is possible that only one or two of the three terms may be empirically significant for a given resource. Similar to the results provided by other studies, Duffield et al. (1992) identified a nonlinear relation of total recreation benefits to flow rate, which shows a concave shape that increases with flow to a point but then decreases for further increases in flow (see figure 4.7). For example, at very low flows rapids are not a challenge for whitewater boaters, at moderate flows floating quality improves, but at very high flows the rapids become washed out or too dangerous. This concave relation applies to all instream recreation activities, but the flow levels at which recreation quality is maximized differ by activity (Brown et al. 1991).

The analytical form of the demand curve for IRA adopted in this model (4.25) was from the aforementioned study and is shown in figure 4.7. The marginal recreational value of instream flow, expressed in dollars per million cubic meter ($/Mcm), is assumed to follow a linear decreasing model:

$$b^{IRA}\,[\$/Mcm] = a_5 - b_5 R \qquad (4.25)$$

where R represents the total flow (volume during the time period) passing through the recreation

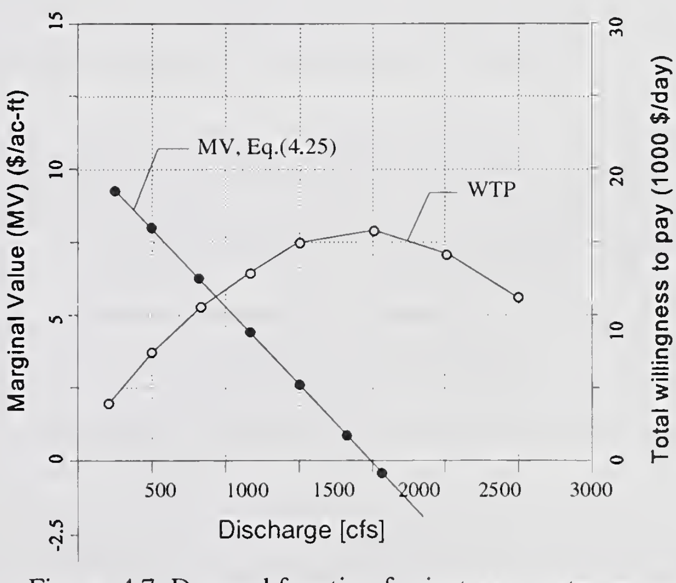


Figure 4.7 Demand function for instream water recreation (1988 dollars) (Duffield et al. 1992).

area and a_5 and b_5 are parameters of the linear function. Because no consumptive water use takes place in the IRA reach (except for evaporation and infiltration losses from the stream, which are typically small and ignored), the same amount of flow R is available downstream from the recreational user (see figure 4.8).

The integral of the demand curve with respect to the instream flow yields the total benefits derived from the IRA river reach:

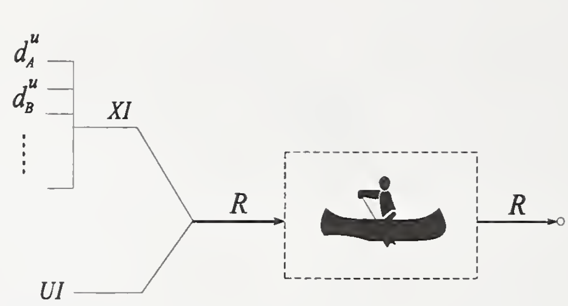


Figure 4.8 In/outflows for an instream recreation area.

$$B_i^{\text{IRA}}\,[\$] = \int_0^{R_i} (a_{5_i} - b_{5_i} z)\, dz \tag{4.26}$$

where z is a dummy variable of integration. Solving the integral in (4.26), the IRA objective function for a single time interval i is:

$$B_i^{\text{IRA}}\,[\$] = a_{5_i} R_i - \frac{b_{5_i}}{2} R_i^2 \tag{4.27}$$

that also indicates that the parameters a_5 and b_5 can vary seasonally.

The total flow R passing through the IWR river reach can be several controlled and uncontrolled flows as depicted in figure 4.8 and expressed mathematically in (4.28), where the controlled inflow term XI is expressed by $d_{A_i}^u, d_{B_i}^u, ..$ to indicate the possibility of multiple controlled variables contributing flows to the recreation area:

$$R_i = f[XI_i, UI_i] = f[(d_{A_i}^u, d_{B_i}^u, ...), UI_i] \tag{4.28}$$

Substituting (4.28) into (4.29), the expanded form of the benefit function is:

$$B_i^{\text{IRA}}\,[\$] = a_{5_i}(c_A^u d_{A_i}^u + c_B^u d_{B_i}^u + ... + UI_i) - \frac{b_{5_i}}{2}(c_A^u d_{A_i}^u + c_B^u d_{B_i}^u + ... + UI_i)^2 \tag{4.29}$$

where $c_A^u, c_B^u, ..$ are the coefficients of the upstream decision variables $d_A^u, d_B^u, ..$ respectively.

Chapter 5

Water Allocation Model

This chapter introduces the method implemented in AQUARIUS V96 to solve a water allocation problem. Following definition and mathematical formulation of the problem, the chapter presents details of the sequential optimization technique that allocates the water throughout the flow network. A list with operational constraints that typically restrict reservoir storage and release volumes is presented. The chapter ends with an accounting of the criteria used to find an initial feasible solution and a discussion of the lengths of the time intervals used for simulating a system's operation.

Problem Statement

The aim in using a limited resource, such as water in a river basin, is to realize the greatest possible value from its allocation over some time period consistent with operational and institutional constraints and with the highest reliability. In AQUARIUS V96, "value" is represented by economic benefit functions that express society's willingness to pay for the various water uses. These benefit functions may be downward sloping and nonlinear. However, the model can be adapted to implement other definitions of value. For instance, the user may wish to reflect a set of priorities such as stipulated by the Doctrine of Prior Appropriation. In this case, the demand functions would be specified as horizontal, with the constant prices of those functions set to represent the seniorities of the demand areas holding the water rights. Allowing for capability of a downward sloping, nonlinear, demand function offers the greatest flexibility to the user.

In modeling an actual allocation problem, it is imperative that problem formulation, particularly the objective function, retain the essential characteristics of the system being modeled. Realistic conceptualization of a water allocation problem typically requires relatively complex model formulations. A common source of this complexity is the nonlinear nature of production and benefit functions. Nonlinear optimization problems are commonly encountered in water resources applications. Besides the nonlinearities introduced in the model by hydroelectric power production, the economic benefits of several different water uses are often best represented by nonlinear benefit functions.

AQUARIUS uses optimization tools to provide the analyst with a way to find the optimal strategies for allocating water among the several uses. The expression "optimal" is used here in a restrictive sense, to indicate a water allocation strategy that is best with respect to the physical and economic criteria specified in a particular model formulation. The model can also analyze alternative operations, the results of which can be compared to aid the decision maker in selecting the most socially desirable policy.

Objective Functional for Optimal Allocation

In AQUARIUS, water allocation throughout a system and for an entire planning horizon is based on a global objective; to maximize the sum of all economic benefits from the instream and offstream water use. The benefit functions for each of the system components were developed in Chapter 4. What remains is to combine those individual benefit functions B into a total benefit function TB that reflects all water uses u in the basin, HPW^{VH}, HPW^{FH}, IRR, M&I, and IRA, and all time periods i in the planning horizon (see also (2.2)). The overall objective is to maximize the total benefit function TB:

$$\underset{\mathbf{x}}{maximize}\ TB(\$) = \sum_{i=1}^{np} \sum_{u=1}^{nu} B_i^u \tag{5.1}$$

where $\mathbf{x}$ denotes the set of control variables, nu is the number of water uses generating revenue in the basin, and np is the number of time periods (optimization horizon).

Equation (5.1) considers only benefits from water use. Costs of water use, such as the cost of operating a hydroelectric plant or of constructing an irrigation canal, are not explicitly considered in the model. The model could be used to evaluate net benefits (i.e., the difference between benefits and costs) by subtracting costs directly from the benefits in the individual benefit functions; essentially making each benefit function a net benefit function.

The problem is to maximize the general nonlinear objective function (5.1), also expressed as $f(\mathbf{x})$, subject to physical, operational, and institutional restrictions such as:

- reservoir storage limitations,
- firm water supply,
- seasonality of water supply,
- firm energy production,
- max/min instream flows, and
- max/min offstream diversions.

Mathematically, these restrictions can be expressed as three types of constraints:

- equality constraints $$\sum_{n=1}^{N} a_{kn} \mathrm{x}_{\mathrm{n}} = r_k \qquad for\ k = 1,2,\ldots,K \tag{5.2a}$$
- inequality constraints $$\sum_{n=1}^{N} a_{kn} \mathrm{x}_{\mathrm{n}} \geq r_k \qquad for\ k = K_{e+1},\ldots,K \tag{5.2b}$$
- bounded variables $$\mathrm{x}_{\mathrm{n}}^{\mathrm{L}} \leq \mathrm{x}_{\mathrm{n}} \leq \mathrm{x}_{\mathrm{n}}^{\mathrm{U}} \qquad for\ some\ of\ the\ x_n \tag{5.2c}$$

where **x** denotes decision variables, N is the total number of decision variables, K_e is the number of equality constraints and K is the total number of constraints. Except for the nonlinear objective function, the problem above is similar to a standard linear programming problem. The function $f(\mathbf{x})$ can be any type of nonlinear function subject to the requirement of being continuous and differentiable.

Solution Method

There are a variety of approaches for solving the above problem, none of which is uniquely superior. The solution technique implemented in AQUARIUS V96 takes advantage of the special case of the general nonlinear programming problem that occurs when the objective function is reduced to a quadratic form and all the constraints are linear. Although a quadratic function is the simplest nonlinear approximation that can be used for a nonlinear objective, it is suitable for solving multi-reservoir optimization problems with hydropower generation (Díaz and Fontane 1989). Furthermore, model development indicated that, because of the particular structure of the water allocation problem, a quadratic approximation of the objective function is advantageous from the computational viewpoint. A quadratic approximation is a close representation of the nonlinear objective function defined in (5.1) and also permits larger, valid changes in the control variables at each step of the solution in comparison to a simple linear approximation. This results in a faster convergence to the optimal solution and lower risk of solution divergence (Díaz and Fontane 1989)

The method approximates the original nonlinear objective function with a quadratic equation using Taylor series expansion and then solves the problem as a quadratic programming (QP) problem. Starting with an initial feasible solution, $\mathbf{x}^o$, the algorithm carries out a Taylor series expansion on the nonlinear objective function around the given initial solution, retaining the first and second order terms to form a quadratic function. The general Taylor series expansion of $f(\mathbf{x})$ truncated beyond the second order terms yields:

$$f(\mathbf{x}) \approx f(\mathbf{x}^o) + \nabla f(\mathbf{x}^o)^T \partial\mathbf{x} + \frac{1}{2}\partial\mathbf{x}^T \mathbf{H}(x^o)\,\partial\mathbf{x} \tag{5.3}$$

where $\partial\mathbf{x}$ is the increment vector and $\nabla f(\mathbf{x}^o)$ is the gradient vector of $f(\mathbf{x})$ measured at $\mathbf{x}=\mathbf{x}^o$. The gradient is computed by the first partial derivatives evaluated at $\mathbf{x}=\mathbf{x}^o$,

$$\nabla f = \mathrm{col}(\partial f/\partial x_1, \partial f/\partial x_2, \ldots, \partial f/\partial x_N) \tag{5.4}$$

The remaining term, $\mathbf{H}(\mathbf{x}^o)$, denotes the Hessian matrix, which is a real matrix whose elements are evaluated at the same point $\mathbf{x}=\mathbf{x}^o$,

$$
\mathbf{H} = \begin{vmatrix} \dfrac{\partial^2 f}{\partial x_1 \partial x_1} & \dfrac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots\cdots & \dfrac{\partial^2 f}{\partial x_1 \partial x_N} \\ \dfrac{\partial^2 f}{\partial x_2 \partial x_1} & \dfrac{\partial^2 f}{\partial x_2 \partial x_2} & \cdots\cdots & \\ \cdots\cdots & & & \\ \dfrac{\partial^2 f}{\partial x_N \partial x_1} & \dfrac{\partial^2 f}{\partial x_N \partial x_2} & \cdots\cdots & \dfrac{\partial^2 f}{\partial x_N \partial x_N} \end{vmatrix} \tag{5.5}
$$

To achieve the optimization, the quadratic approximation (5.3) of the nonlinear objective should conform to the standard form of a quadratic programming problem, as defined by:

$$
\underset{\mathrm{X}}{Max} \left[f(\mathbf{x}) = w + \mathbf{c}^{\mathrm{T}}\mathbf{x} + \frac{1}{2}\mathbf{x}^{\mathrm{T}}\mathbf{Q}\,\mathbf{x} \right] \tag{5.6a}
$$

- subject to the linear constraints $\quad g(\mathbf{x}) = \mathbf{A}\,\mathbf{x} \geq \mathbf{r}$ (5.6b)
- and nonnegativity conditions $\quad \mathbf{x} \geq \mathbf{0}$ (5.6c)

where w is a scalar, the **c** and **r** vectors have known components, **A** is the matrix of constraint coefficients, and **Q** is a square matrix of dimension (NxN). The components of w, the **c** vector, and the **Q** matrix can be obtained by equating (5.3) and (5.6a). Details of the derivation are in Díaz and Fontane (1989). The final expressions are:

$$
w = f(\mathbf{x}^{\circ}) - \sum_{i=1}^{N} D_i \mathrm{x}_i^{\circ} + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \mathrm{H}_{i,j}\,\mathrm{x}^{\circ}\mathrm{x}_j^{\circ} \tag{5.7}
$$

$$
c_i = D_i - \sum_{j=1}^{N} H_{i,j}\,\mathrm{x}_j^{\circ} \qquad \text{for } i = 1,2,\ldots.,N \tag{5.8}
$$

$$
\mathrm{q}_{i,j} = \mathrm{H}_{i,j} \qquad \text{for } i = 1,2\ldots,N \text{ and } j = 1,2\ldots,N \tag{5.9}
$$

where the only new term is D_i, which equals the i^{th} element of the gradient vector ($\partial f/\partial \mathrm{x}_i$). Once the equivalent **c** vector and **Q** matrix have been explicitly defined, a standard QP code can be used to solve for the set of **x** values that maximizes the objective function.

An efficient QP code is a basic requirement for the success of the proposed solution method. The routine QPTHOR, based on the General Differential Algorithm (Wilde and Beightler 1967) and further developed by Leifsson and Morel-Seytoux (1981), is used in AQUARIUS V96.

The optimal solution obtained by standard QP is only true for the approximated objective function (5.3). Because the optimal values of the variables may differ from the initial values upon which the approximation of the nonlinear quadratic objective function was based, it is necessary to repeat the process using the new values for the set of variables as the starting point for the next round of the sequential solution. A succession of these approximations is performed until the solution of the quadratic programming problem reaches the optimal solution, which is when successive optimal values do not differ by more than the stipulated tolerance limit, or when the maximum limit on the number of iterations is reached. Figure 5.1 illustrates the sequential procedure of successively solving quadratic programming problems, known as Sequential Quadratic Programming (SQP). The SQP approach implemented in AQUARIUS is an extension of the work reported by Díaz and Fontane (1989) and Díaz et al. (1992).

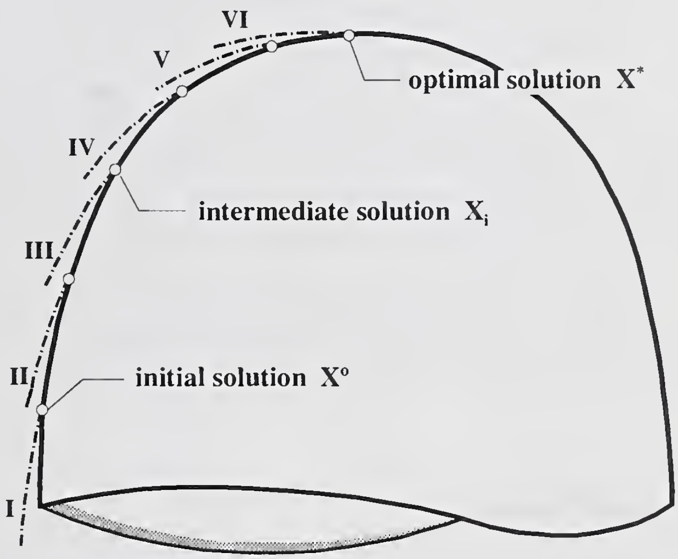


Figure 5.1 Sequential maximization of a concave objective function by Sequential Quadratic Programming (SQP) (Díaz and Fontane 1989).

Linear approximation of the nonlinear objective function (5.1), using sequential linear programming (SLP), is also a viable alternative to solve the water allocation problem with AQUARIUS. SLP convergency to the optimal solution is only possible when a step-bound solution scheme in the control variables is implemented (Palacios Gomez et al. 1982). This causes SLP to have a slower rate of convergency toward the optimal solution than SQP (Díaz and

Fontane 1989). Morever, optimal values tend to be slightly lower with SLP than with SQP, with the difference more pronounced as the complexity of the water allocation problem increases. Nevertheless, for small flow networks or when only some of the variables in the objective function are nonlinear, SLP is an efficient algorithm.

Objective Function Differentiation

The gradient vector in (5.3) and the Hessian matrix in (5.5) require computation of first, second, and second-cross partial derivatives of the global objective function with respect to the decision variables $\mathbf{x}$. For a water allocation problem, which may involve many control variables and complex objective functions, a finite difference scheme may appear to be an expeditious procedure for the differentiation. However, numerical differentiation is inherently inaccurate, particularly for high-order differentiation.

In contrast, differentiation via calculus provides the exact results for partial derivatives of any order. Accuracy of the computations is important if the QP algorithm is to arrive at an exact solution. Calculus also reduces the computation time drastically. The algebraic derivation of the partial derivatives using calculus was an involved and time consuming task as the model had to consider all possible water uses generating revenue in the basin and all possible ways in which they could interrelate within a flow network. The derivative formulas are in Appendix A. Once the formulas are completed, assembling the gradient vector and Hessian matrix for any user-defined network topology is greatly eased, partly due to the object-oriented modeling framework adopted. This subject, called the mathematical connectivity of the network, is covered in Chapter 6. The derivatives in Appendix A are organized by water user first. For each type of benefit function, several partial derivatives are considered based on all possible control variable roles in a flow network.

Operational Restrictions

As indicated, the model maximizes total return over a planning horizon, subject to all required operational restrictions. These restrictions, or constraints, represent physical limits in the operation of reservoirs, powerplants, diversion structures, and other system components. We present a list of basic operational constraints that are essential for the proper simulation of a water system.

Given the multi-site and multipurpose nature of the formulation, it is necessary to adopt a notation to help us distinguish between variables associated with different system components (objects) and the relative location of the system components in the flow network. The following notations complement those in the Storage Reservoirs section of Chapter 3:

d = decision variable (controlled flow)
d_M = upper bound of decision variable
d_m = lower bound of decision variable
NF = natural (uncontrolled) flow
S = reservoir storage
S^o = initial reservoir storage
S^f = final reservoir storage
S_M = maximum reservoir operational storage
S_m = minimum reservoir operational storage
E = net reservoir evaporation
L = reservoir spillage
MR = mandatory reservoir release
IF = instream flow
IF_M = maximum instream flow
IF_m = minimum instream flow
OF_M = maximum offstream water supply
OF_m = minimum offstream water supply

Because of the generalized model character, the constraint equations presented below are not for any specific topology of a flow network, but rather are generally formulated consistent with the object-oriented approach used throughout this model. Each system component (i.e., each model object) may have one or more constraints associated with it. Constraint equations can be either equalities or inequalities depending on the type of object and the nature of the constraint being imposed.

The superscript u denotes a variable that originates in the portion of the network upstream from the object under consideration. The superscript s denotes a variable that outflows from the same object being analyzed (figure 5.2). The subscript i denotes the time period. For instance, i=3 indicates the third time interval of the optimization horizon. For all variables except reservoir storage, the index i indicates the variable value during the time interval such as inflow to a reservoir during the third time interval. When referring to reservoir storage, the index i should be interpreted as the storage at the end of the time period i.

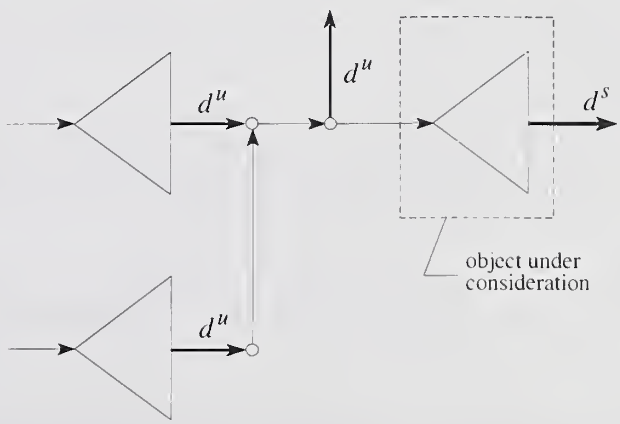


Figure 5.2 Sets of decision variables in relation to the object under consideration.

There may be multiple sources of controlled and uncontrolled flows converging at a given system component. For example, figure 5.2 depicts a network with three d^u sets that are the controlled inflows to the downstream reservoir. Although not shown in the figure, this may also occur with upstream mandatory releases, spillages, and natural flows. To simplify the notation, this situation is denoted in the constraint equations below by underscoring the symbol representing the variables cited above. The signs of the decision sets d^u and d^s (i.e., the sign of the coefficients of the decision sets) and all other terms in the constraint equations are automatically given by the model. In general, signs will be positive or negative depending on whether the variable of interest contributes flow to or removes flow from the object under consideration (this is the water-balance concept). More details concerning the mathematical connectivity of system components are in Chapter 6, Mathematical Connectivity of System Components.

All restrictions are written using the canonical form of expressing a mathematical programming problem. The left-hand sides of the constraint equations are the decision variables (unknown terms). The right-hand sides contain the uncontrolled (known or assumed known) terms. The system restrictions available in AQUARIUS V96 are:

Maximum reservoir storage: ensures that the storage at the end of any time period i does not exceed the reservoir's maximum operational capacity (i.e., $S_i \leq S_M$) for $i = 1, 2, ..., np$, where np is the total number of periods:

$$\sum_{k=1}^{i} \underline{d}_k^u + \sum_{k=1}^{i} \underline{d}_k^s \geq (S^o - S_M) + \sum_{k=1}^{i} (\underline{MR}_k^u + \underline{NF}_k^u + \underline{L}_k^u) - \sum_{k=1}^{i} (MR_k^s + L_k^s + E_k^s) \tag{5.10}$$

Minimum reservoir storage: ensures that the storage at the end of any time period i does not fall below the reservoir's minimum active capacity (i.e., $S_i \geq S_m$) for $i = 1, 2, ..., np$:

$$\sum_{k=1}^{i} \underline{d}_k^u + \sum_{k=1}^{i} \underline{d}_k^s \geq (S_m - S^o) - \sum_{k=1}^{i} (\underline{MR}_k^u + \underline{NF}_k^u + \underline{L}_k^u) + \sum_{k=1}^{i} (MR_k^s + L_k^s + E_k^s) \tag{5.11}$$

Reservoir final storage: imposes a final reservoir storage level S^f. This prevents the model from generating extra benefits at the expense of depleting a reservoir's storage at the end of the optimization horizon:

$$\sum_{k=1}^{i} \underline{d}_k^u + \sum_{k=1}^{i} \underline{d}_k^s = (S^f - S^o) - \sum_{k=1}^{np} (\underline{MR}_k^u + \underline{NF}_k^u + \underline{L}_k^u) + \sum_{k=1}^{np} (MR_k^s + L_k^s + E_k^s) \tag{5.12}$$

Maximum instream flow: ensures that flows do not exceed a specified maximum, as may be required for instream water-related activities such as recreation or environmental protection (i.e., $IF \leq IF_M$):

$$\underline{d}_i^u \geq -IF_{M_i} + (\underline{MR}_i^u + \underline{NF}_i^u + \underline{L}_i^u) \qquad for\ i = 1,..,np \tag{5.13}$$

Minimum instream flow: ensures that flows do not fall below a specified minimum, as may be required for instream water-related activities such as recreation or environmental protection (i.e., $IF \geq IF_m$):

$$\underline{d}_i^u \geq IF_{m_i} - (\underline{MR}_i^u + \underline{NF}_i^u + \underline{L}_i^u) \qquad for\ i = 1,..,np \tag{5.14}$$

Diversion node: ensures that the flow diverted from a diversion node does not exceed the incoming flow:

$$\underline{d}_i^u + \underline{d}_i^s \geq -(\underline{MR}_i^u + \underline{NF}_i^u + \underline{L}_i^u) \qquad for\ i = 1,..,np \tag{5.15}$$

Maximum offstream flow: ensures that flow supplied to an irrigation or urban area does not exceed a specified maximum (i.e., $OF \leq OF_M$):

$$\underline{d}_i^u \geq -OF_{M_i} + (\underline{MR}_i^u + \underline{NF}_i^u + \underline{L}_i^u) \qquad for\ i = 1,..,np \tag{5.16}$$

Minimum offstream flow: ensures that flow supplied to an irrigation or urban area does not fall below a specified minimum (i.e., $OF \geq OF_m$):

$$\underline{d}_i^u \geq OF_{m_i} - (\underline{MR}_i^u + \underline{NF}_i^u + \underline{L}_i^u) \qquad for\ i = 1,..,np \tag{5.17}$$

Seasonality of water demand: ensures that water deliveries to demand areas follow a user defined seasonal pattern (see figure 3.7). The constraint is applicable for time periods $i = 1, 2, ..., npc$, where *npc* denotes the total number of periods within the annual cycle ($npc=12$ for monthly intervals). To activate this constraint, the optimization horizon should encompass entire annual cycles:

$$\underline{d}_i^u - \alpha_i \sum_{k=1}^{npc} \underline{d}_k^u = -\underline{UI}_i^u + \alpha_i \sum_{k=1}^{npc} \underline{UI}_k^u \qquad for\ \ 0 \leq \alpha_i \leq 1\ \ and\ \sum_{k=1}^{npc} \alpha_i = 1 \tag{5.18}$$

Annual firm water supply: enforces firm levels of annual supply to offstream demand areas, where *AFW* represents the contracted annual volume and *npc* denotes the number of periods within the annual cycle. To activate this constraint, the optimization horizon should encompass entire annual cycles:

$$\sum_{k=1}^{npc} \underline{d}^u_k \geq AFW - \sum_{k=1}^{npc} \underline{UI}^u_k \tag{5.19}$$

The sequential-approximation algorithm SQP is well suited for solving this water allocation problem, which includes only linear constraints. However, the algorithm can be extended to problems with nonlinear constraint functions by using linear approximations of the nonlinear constraint equations (not included in AQUARIUS V96).

In the formulation of the water allocation problem, the model automatically includes the restrictions in equations (5.10) through (5.12). The user has the option to enable or disable restrictions by toggling on/off the corresponding check-boxes in the input dialog boxes. Even a mid-size network may include hundreds of constraint equations, depending on the number of system components and the length of the optimization horizon.

Controlling flows and storage are not the only restrictions commonly found in the operation of real-world water systems. For instance, the two restrictions below, which are related to hydropower production, will be incorporated into the model in a future version:

> **Hydropower Firm Energy**. This constraint would be used to guarantee that a reservoir/powerplant subsystem were operated to guarantee the delivery of a preestablished amount of electrical energy (megawatt-hours) with assured availability to the electrical network. Firm-energy levels could be demanded at each powerplant separately or from the system of powerplants as a whole depending on ownership and agreements among the electrical facilities within the river basin.
>
> **Maximum Energy Sale**. This constraint would allow the analyst to impose a maximum value to the total amount of energy, per simulation period or over the year, that the hydro system could sell to the electrical grid. Nevertheless, in AQUARIUS V96 the user can limit the powerplant discharges as an approximate way to control the electrical output generated.

Modeling Uncontrolled Releases

The term uncontrolled releases *UR* (see figure 3.1 and equation (3.3)) refers to the volume of water evaporated from the reservoir *E* and the spillway releases at the dam *L*. The model uses a simplified approach for handling such losses, which treats *E* and *L* as constants for each solution of the QP problem. These constant values are based on the result of the previous QP solution. The assumption of *E* and *L* as constants is clear in constraint equations (5.10) through (5.12), where all the known (or assumed known) terms are on the right-hand side (RHS). By treating evaporation and spills as constant terms in the formulation, the nonlinear relationships between

evaporation-storage and outflow-storage, typical of the reservoir dynamics, are omitted, yielding a strictly linear set of constraints.

However, moving reservoir losses to the right side of the constraint equations (see (5.10), (5.11), and (5.12)) gives no control to the model over those outflows, causing E to assume a passive role in the optimization. An alternative formulation of reservoir evaporation is when evaporation losses are included explicitly in the formulation of the model (see Appendix C). This more elaborate formulation (to be implemented in a future version) will give the model the capability to anticipate periods with extreme evaporation losses and affect the operation of reservoirs accordingly.

Simplified Modeling of Reservoir Evaporation

The consequences of assuming known values for evaporation when solving the QP problem are: 1) the optimal reservoir storage provided by each sequence of the SQP process (i.e., by each individual QP solution) must be adjusted using an iterative reconciliation procedure to find accurate estimates of E; and 2) because new values of E were obtained from the previous step, all RHSs must be recomputed before going to the next optimization sequence. The iterative reconciliation procedure, performed after each QP solution, works as follows: 1) based on the last optimal state of the reservoirs, evaporation losses are computed; 2) because the new evaporation losses probably differ from the old ones (those adopted when solving the last QP problem), new (adjusted) reservoir storages will result; 3) based on these adjusted reservoir storages, evaporation losses are recomputed; 4) step 3 is repeated until S and E values are reconciled for all reservoirs; that is, until the difference in storages between two consecutive iterations is below a given threshold, which is a user defined parameter.

Such computations raise the following question: Will a QP solution that is feasible with respect to its constraint set (i.e., the set containing E from the previous QP solution) become infeasible in the new constraint set? To answer this question, consider the hypothetical case of a reservoir storage trajectory constrained by upper and lower bounds in figure 5.3. The thin solid line (old solution) represents the optimal trajectory derived from the prior QP solution, as adjusted using the iterative reconciliation procedure. This solution is used as the initial feasible solution for the next QP solution, which yields a new storage trajectory represented by the dashed line (new solution). For the portion of the trajectory that is successively moving toward the lower bound (left side of the graph), the water losses during computation of the new QP solution (from the old solution) are larger than the true losses that would occur at the new solution storage level (i.e., the surface area of the reservoir decreases). After evaporation losses and the storage level agree the new optimal trajectory will recede inward because lowering the evaporation loss will increase the remaining storage. This is shown by the thick solid line (adjusted new solution) on the graph.

Similarly, for the portion of the trajectory that moves closer to the upper bound (right side of the graph), the water losses during computation of the new QP solution are smaller than the true losses that would occur at the new storage level because the new storage level is higher than that

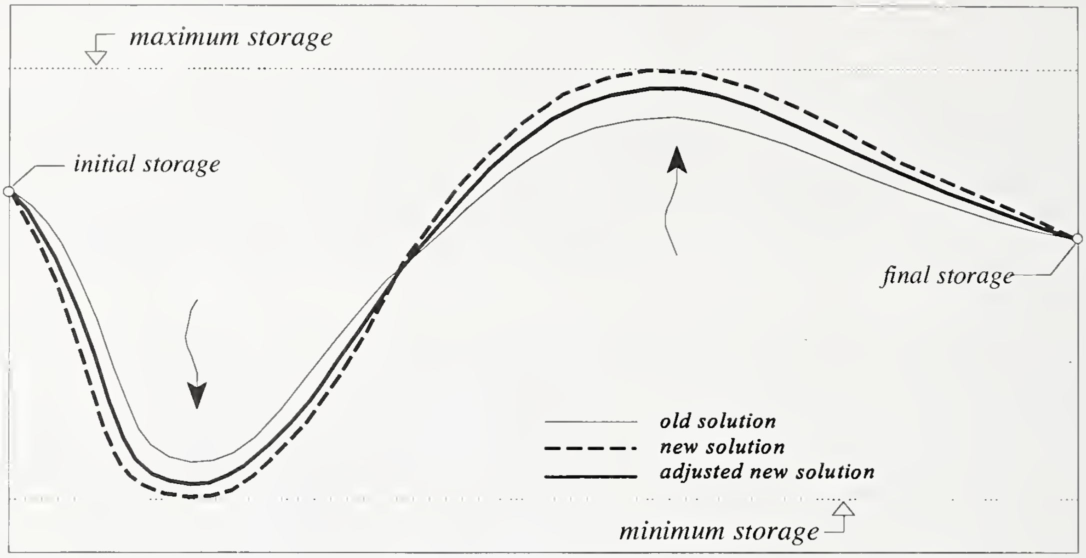


Figure 5.3 A storage trajectory adjusted for evaporation losses between Quadratic Programming solutions.

of the old solution from which the assumed losses were taken. After bringing storage and losses into agreement using the iterative procedure, the new optimal trajectory will recede inward as indicated by the thick solid line. These two cases indicate that the prior optimal solution will remain feasible to the new constraint set, allowing the sequential optimization procedure to continue. The same conclusion is reached when analyzing a sequence of storage trajectories that tend to depart from upper and lower reservoir bounds.

Analysis of this hypothetical case shows that the iterative process by which evaporation losses are evaluated, except under very unusual circumstances, should not cause infeasible solutions as the storage trajectory progresses toward optimality. Furthermore, the sequential nature of the SQP algorithm allows for the convergence of uncontrolled outflows toward their exact values.

Simplified Modeling of Reservoir Spillages

Because of the inherent nature of the objective function, characterized by decreasing marginal returns as a function of increasing releases, reservoir spillages are minimized subject to specified constraints. The model drives a solution containing spills toward a different and improved solution where excess water is reallocated over the whole operational horizon in favor of a more profitable water allocation. In this manner, spills are converted to the extent feasible into storage, giving the optimization scheme the opportunity to transform the excess volume of water into economically beneficial reservoir releases during other time periods.

This simplified approach is further justified if we realize that: 1) it has the advantage of reducing the dimensionality of the optimization problem; 2) spillages typically occur infrequently during the life of a project; and 3) a monthly time-interval of simulation precludes any detailed consideration of a spillway control structure.

Search for a Feasible Solution

The method of solution described previously requires an initial feasible solution (IFS) to start the optimization process. This is an important requirement, especially for large, highly constrained water systems. For small flow networks and short periods of analysis, it is reasonable to expect the user to provide the required IFS. More complex situations, such as those that the current model is designed for, require a modeling approach.

AQUARIUS contains two different approaches to obtaining an initial feasible solution. The first is an expeditious flow cascading method with some limitations. This approach should provide an IFS for most systems and flow conditions. The second approach is a more involved but robust method based on the same concepts as the "Two-Phase Method of Linear Programming." If the flow cascading method failed to provide the IFS, this second approach would complete the task. Both methods are described below.

If, due to intricacies of the flow network, the model is unable to find an IFS that fully complies with the originally stipulated constraints, the analyst may relax restrictions that impede finding a initial solution. After an IFS is found with the relaxed set of constraints, the analyst will have an opportunity to gradually steer the water allocation process in the appropriate direction during the optimization process. This is possible because the water allocation problem is formulated using economic variables that can be manipulated during the sequential optimization without affecting the feasibility of any intermediate solution. Once the necessary level of flow is attained as a result of the modified economic conditions, the optimization can be interrupted, the relaxed constraints reinstated, and the original price structure restored, allowing the model to continue with the optimization until the final optimal solution is reached.

Flow Cascading

The flow cascading approach consists of routing the natural flows originating in the basins downstream through the flow network. This is done without any operational rules. The routing algorithm starts at each water source (i.e., a natural flow basin object) and moves downstream through the network always following the main water course. Computationally, this is a pass of the algorithm. As the water travels downstream in the network, it satisfies minimum instream and offstream demands, provided that enough water is available. A demand area that is only partially satisfied after the first pass may become fully satisfied after the second or third pass, depending on the number of water sources being routed through that specific network component. For example, in figure 5.4 only two passes of the algorithm (I and II) provide water

to the offstream irrigation area since the third water source is located downstream from the demand zone.

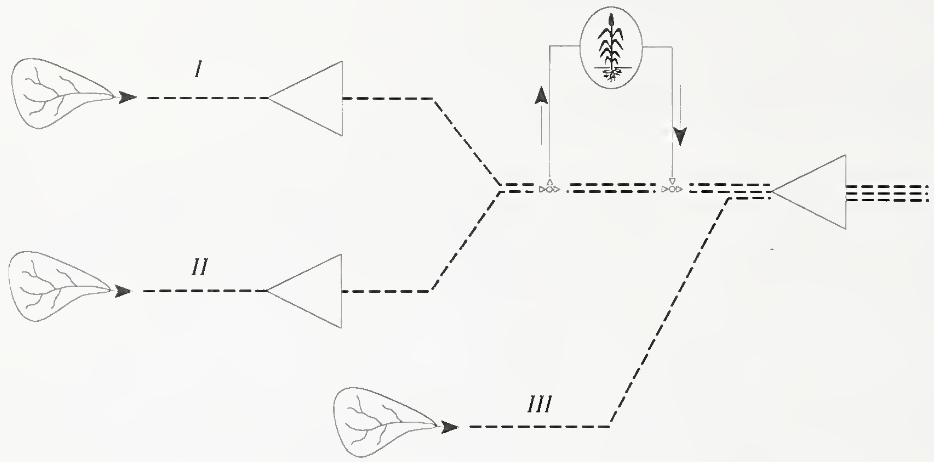


Figure 5.4 Cascading flows through a network to find an initial feasible solution.

Inflows to a reservoir are passed downstream from the structure and used to satisfy reservoir demands. Minimum flow requirements are satisfied first, and any remaining water is arbitrarily assigned to any of the water users connected to the reservoir until maximum capacity is reached. This occurs without any regulation of reservoir storage. However, for very large inflows and when the capacity of the demand areas to accept flow is exceeded, the algorithm is forced to regulate reservoir storage by pre-emptying the reservoir with respect to the period with excessive inflows. If storage regulation alone is unable to contain the inflows, the reservoir is forced to spill.

The cascading approach may find an IFS, depending on how tightly constrained the water system is given water availability in the basin. In general, normal and wet flow conditions are best for a successful run. If flow conditions are very dry, relaxation of constraints, such as specific minimum flow requirements, increases the chance of a successful run. The cascading approach always assumes that the final storage of a reservoir S^f will equal the selected initial storage S^o.

Phase 1 of the Two-Phase Method

To find an initial feasible solution as required by the General Differential Algorithm (QPTHOR), it is possible to use the same procedures as the Simplex Method of Linear Programming (LP) uses for the same purpose. Some of these procedures involve use of artificial variables to obtain an initial basic feasible solution to a slightly modified set of constraints (although for QPTHOR, the initial solution does not need to be basic). From among these procedures, we adopted a well known algorithm, the Two-Phase Method, for use in AQUARIUS. Below is a brief description of the method. For a complete description see Bazaraa and Jarvis (1977).

During the denominated Phase 1 of the Two-Phase method, the restriction set in (5.6b) is changed by adding a vector of artificial variables $\mathbf{x_a}$, leading to the system of equations $\mathbf{Ax+x_a=r}$, $\mathbf{x\geq 0}$, $\mathbf{x_a \geq 0}$. By construction, $\mathbf{A}$ contains an identity matrix associated with the artificial vector. This gives an immediate basic feasible solution of the new problem, namely, $\mathbf{x_a=r}$ and $\mathbf{x=0}$. However, even though an initial feasible solution now exists, the problem has in effect been changed. To get back to the original problem, the artificial variables should be forced to zero to attain feasibility in the original problem because $\mathbf{Ax=r}$ if and only if $\mathbf{Ax+x_a=r}$ with $\mathbf{x_a=0}$. The modified LP problem solved during Phase 1 with the basic feasible solution $\mathbf{x=0}$ and $\mathbf{x_a=b}$ is:

Minimize $\mathbf{1\ x_a}$ (5.20a)

Subject to ... $\mathbf{A\ x + x_a = r}$ (5.20b)

$\mathbf{x, x_a \geq 0}$ (5.20c)

where, since the objective function is replaced by an auxiliary function, it is inconsequential that the original objective function (5.6a) is nonlinear. Then, the Simplex Method is used to minimize the sum of the artificial variables over the feasible region for the revised problem. The solution for Phase 1 should have all the artificial variables equal to zero so the solution is also feasible for the original problem. At the end of Phase 1 we get either a basic feasible solution of the original problem or $\mathbf{x_a=0}$, which implies that the original problem has no feasible solution.

Since there is no objective function driving the Phase 1 solution to any particular state of the system, the constraint set (5.20b) basically defines the IFS. The same set of constraint equations used to restrict the SQP optimization (see Chapter 5, Operational Restrictions) is also used for (5.20b). An obstacle during the computation of the constraint set (5.20b) is the dependance of some of the RHSs on uncontrolled spills and evaporation losses, which are unknown before-hand (as discussed in Chapter 5, Modeling Uncontrolled Releases). To start the computation process, evaporation losses are initialized to some reasonable values, whereas spills are initially assumed equal to zero. In this manner, the model is forced to find an IFS free of spills.

However, when the inflows are characterized by very wet periods, the above restriction may prevent a solution that satisfies the set of constraints. When this occurs, the Phase 1 problem is reformulated assuming positive semi-infinite reservoirs and the problem is resolved. Then, water held artificially in storage (i.e., above the maximum operational storage capacity) is transformed into spills from the reservoir. The last estimates of spills are used to reformulate the Phase 1 problem for a third time, which finally yields the required IFS.

Although this approach is more involved than the simple flow cascading procedure, the efficiency of the LP algorithm and the numerical stability of the Two-Phase Method ensures a much better chance of finding an IFS under all flow conditions.

Time Intervals of Analysis

The analysis performed using AQUARIUS defines the length and number of time intervals to be considered. For example, although an analysis at the monthly level may be sufficient during the planning of a water resources project, simulating the actual operation of a river system may require weekly or daily time intervals. The length and number of time intervals of an analysis is also affected by the availability of time-dependent data, especially flow data, required to run the model and by execution time.

Modeling water allocation in a basin may require considerable detail for some specific water uses. An example is instream flow to maintain an activity such as whitewater rafting. This water use is highly seasonal, occurring only a few months of the year, and has a nonuniform demand within the recreation season. The demand for whitewater rafting changes within the week because use is greater on the weekends than during weekdays. Detailed consideration of the water supply for this activity in competition with other water uses in the basin may justify modeling system operation at a daily time step. Moreover, if the reservoir system contains enough storage to carry over from one year to the next, then a multi-year operation must be modeled. In this manner, benefits of storing excess water during wet years for future releases during dry years can be realized. Contrarily, if the reservoirs fill practically every year, the period of analysis can be limited to a single year, known as within-year operation. Each system will present different characteristics and will impose different time step and time horizon requirements for optimization.

AQUARIUS is envisioned to simulate the water allocation in a basin using any time interval of analysis including daily, weekly, and monthly time intervals. Moreover, the model is envisioned to operate under time intervals of uniform as well as nonuniform length. As an example of the latter, consider a one-year optimization horizon subdivided into the first 7 days (short-term operation), the following 3 weeks (medium-term), and the remaining 11 months (long-term). This partition of the within-year operation into intervals of unequal length may coincide with the way inflows to the system are forecasted. Because of limitations in the graphical user interface (GUI) for entering data in the present version of model, only monthly time intervals are accepted. A future version of the model will include the necessary GUIs to work at shorter time intervals.

AQUARIUS can be used in a full optimization mode for general planning purposes or in a quasi-simulation mode with restricted foresight capabilities. For the latter, the model distinguishes between the period of analysis used to specify the length of the whole segment of time for which the model will simulate the allocation of water in the basin and the optimization horizon, which is used to specify how far into the future the model should look to build the optimal operational policies.

Optimizing the operation of a complex water system for an extended period of time (e.g., a period equivalent to the system's economic life, as in an analysis performed for planning purposes) is computationally impractical given typical computing capability. AQUARIUS offers an alternative approach, termed a quasi-continuous optimization, that allows the user to study the

response of a system for a very long time period (e.g., several decades) in a computationally manageable manner. The approach decomposes the full solution of the problem into many consecutive overlapping solutions. Figure 5.5 illustrates this approach for a monthly operation where the overlapping solutions are each three years.

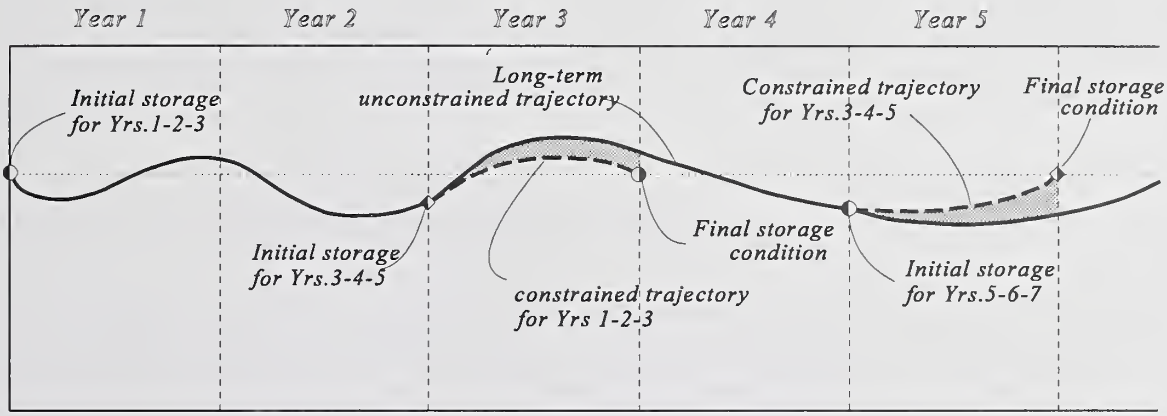


Figure 5.5 Scheme of quasi-continuous optimization.

The solid line of figure 5.5 represents a hypothetical optimal reservoir storage trajectory resulting from optimizing the system operation for many years. In this example, identical initial and final storage conditions are imposed, indicated by the horizontal dotted line. The model begins by optimizing over the first three years (i.e., using an optimization horizon equal to 36 months). The ending storage for this three-year solution must equal the final storage constraint, as indicated by the dashed line showing the end of the three-year trajectory. The model disregards the results from the operation for the third year, saving the state of the system for years one and two. When the next set of three years is optimized, the model adopts, as the initial state of the reservoir, the optimal storage found at the end of the second year of operation. The cycle continues for the total number of years with data available. In figure 5.5 it is assumed that the effect of the imposed final storage does not extend farther back than one year; that is, the dashed and full lines only depart from each other during the third year of operation (filled area). Repeated testing shows that this is often a reasonable estimate, although the best length for the overlapping period should be determined for each specific system. This procedure of quasi-continuous optimization is believed to circumvent the sometimes undesirable effect of the final boundary conditions on reservoir operation. In addition, running the model with restricted foresight capabilities, for example, with an optimization horizon equal to only one or two time periods, would generate operation policies close to what an operator of a real-world system would produce. The optimization model will behave very much like a simulation model. Running AQUARIUS under these conditions requires the addition of targets for reservoir storages as part of the data input requirements.

Chapter 6

Symbolic Modeling of River Basin Systems

This chapter documents the basic architecture of the AQUARIUS software and discusses the advantages of using an object-oriented programming (OOP) framework for modeling the hydraulic and mathematical connectivity of the flow network components. Comprehension of the material in this section is not essential for using AQUARIUS, and is helped by reading Chapter 7.

Object-Oriented Programming Framework

Earlier computer models for solving water resource problems have used algorithmic computer languages such as FORTRAN. Although these languages are well suited for numerical or algorithm-oriented models, they lack the flexibility to allow for alterations, additions, or deletions of flow network components. Current research has overcome this problem by using an OOP language, specifically C++. Objects are the building blocks of an OOP. An object contains properties that communicate with other objects. In turn, object behavior is controlled by methods, which are the rules and algorithms that tell an object how to act on the data it receives in its input slots. Objects may inherit both data (properties) and behavior (methods) from other higher-level objects.

Water systems are ideal candidates to be modeled using an object-oriented framework. A water system may include different types of water components, including reservoirs, powerplants, diversions or junction points, irrigation areas, environmentally-sensitive river reaches, etc, which can be interpreted as objects of a flow network in which they interact. AQUARIUS models each component or structure of the water system as an equivalent node or object in the programming environment. In modeling terms, a physical link (e.g., a river reach) connecting two system components becomes an outflow slot of the upstream object connected to an inflow slot of the downstream object.

The user interacts with the model through a graphical user interface (GUI) that allows the analyst to readily create the river basin network of interest. This is a simple task due to the inherent capability of the object oriented paradigm for graphical representation. During the creation of the flow network, each system component (object) corresponds to a graphical network node. These nodes are represented by icons, which are a pictorial representation of the object. By dragging one of these icons from the menu, the model creates an instance of the object on the screen. This procedure also allows the user to connect graphically the input slots of this object with the output slots of one or more objects. By clicking on the icon, the object displays data slots for input and output and also allows the user to visually inspect for incorrect or missing data. Details on the use of the GUI are in Chapter 7.

The object-oriented terminology and formats for class diagrams used in this document are based on Booch's notation (Booch 1994). The following terms are used in this report:

Object-Oriented Programming (OOP): a programming method in which programs are organized as cooperative collections of objects that represent an instance of some class, and whose classes are all members of a hierarchy of classes united via inheritance relationships.
Class: a set of objects that share a common structure and a common behavior.
Object: an instance of a class.
Instantiation: a new object created from a class.
Hierarchy: a ranking or ordering of abstractions.
Inheritance: a mechanism of hierarchy in which one class shares the structure or behavior defined in one or more classes; there is single or multiple inheritance.
Aggregation: a mechanism of hierarchy wherein a class mimics the behavior of one or more classes by embedding their instances.
Polymorphism: the property of an object, achieved through either inheritance or aggregation, through which it represents objects of many different classes.
Persistence: the property of an object through which its existence transcends time and/or space; the object becomes capable of existing past the lifetime and address space of its creator (e.g., hard disk).
Runtime Type Identification: the property of an object through which it stores the identity of its class so that it is capable of identifying its class type when queried.

Software Architecture

Top-Level Class Diagram

The overall design of AQUARIUS is depicted by the top-level class diagram in figure 6.1. As indicated in the figure, all classes implemented in AQUARIUS are organized into three basic class categories:

- Network Worksheet (NWS),
- Water System Components (WSC), and
- Water System Links (WSL).

AQUARIUS is built upon Microsoft Foundation Classes (MFC), which are a set of reusable classes that provide, with minimal overhead, numerous important functions to software applications written for Microsoft Windows. For instance, MFC supplies built-in support for run-time class identification through the class called *CObject*. Because most classes used in AQUARIUS are derived from *CObject*, they can take advantage of its run-time class identification property.

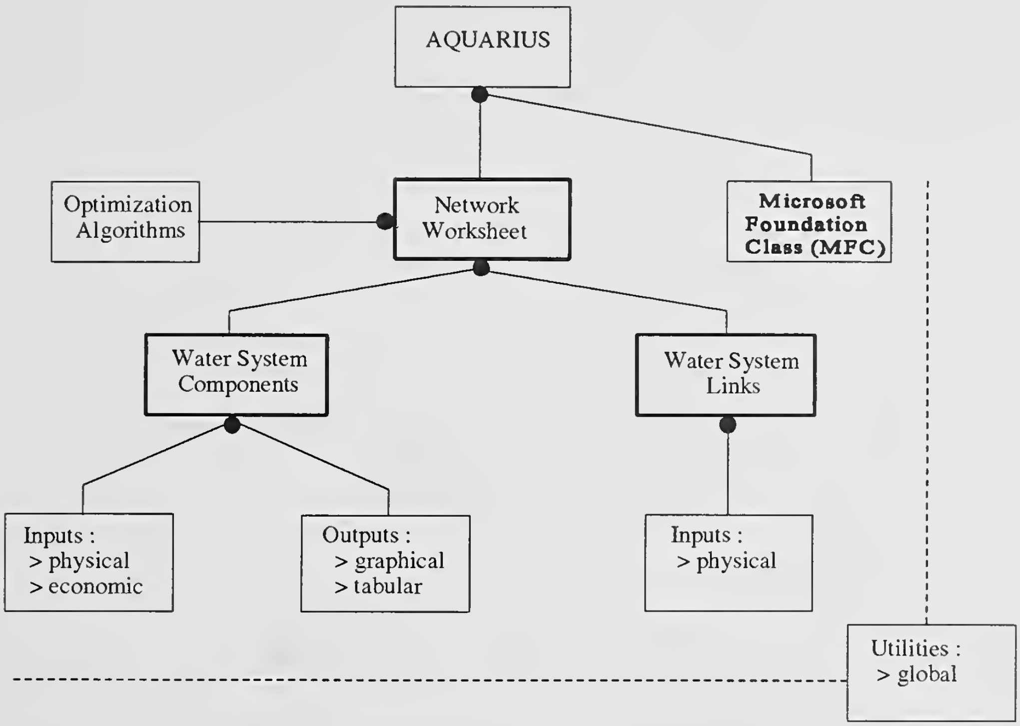


Figure 6.1 AQUARIUS top-level class diagram.

MFC provides encapsulation over the traditional message handling of Microsoft Windows, which relies heavily on callbacks and long switch statements. In projects using MFC, the messages are routed to the member functions of the implemented classes. MFC implements code for the Document-View architecture of the software, which is required for applications like AQUARIUS where data are stored and rendered in some format on the screen. MFC also has the capability to use Multiple Document Interface (MDI) windows for rendering data in more than one window.

Another important service provided by the class *CObject* is object persistence, which is implemented using serialization. Serialization is writing or reading an object to or from a storage medium such as a disk file. In addition, MFC provides built-in support for Object Collection classes to manage a group of objects. The supported collection classes are:

List: an ordered, nonindexed list of elements, implemented as a double linked list; has a head and a tail, and adding or removing elements from the head or tail, or inserting elements in the middle, is very fast; implementation facilitates its use as a stack or a queue.
Array: a dynamically sized, ordered, and integer-indexed array of objects.
Map: a collection class that associates a unique key object with a value object (figure 6.2).

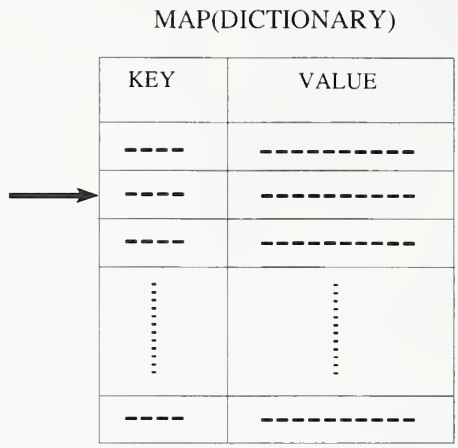


Figure 6.2 Sketch of a map class.

MFC provides a high degree of portability. The application can be easily ported to various operating systems like Windows 3.1, Windows 95, Machintosh, Unix, and different versions of Windows NT.

Class Categories

Network Worksheet

The Network Worksheet (NWS), one of the basic classes of AQUARIUS, is composed of two classes that are responsible for storing and rendering to the screen data associated with any flow network. The document class for the NWS, called *CntflwDoc*, was derived from the class *CDocument* contained in the MFC. The document is a data object that the user interacts with during editing sessions; for instance, during the creation or alteration of a flow network. The view class, called *CntflwView*, is derived from the class *CScrollView*. The view, the user's window to the data, specifies how the user sees the document's data and interacts with it. Class diagrams for the two classes are shown in figure 6.3.

Data pertaining to the network, which is also an object, are stored in the document, and the objects themselves are rendered onto the view window. Most of the important functions of the software, such as user interaction, data persistence, and optimization algorithms, are routed to the individual objects via the message handling facility of the view window. The document portion of the NWS (class *CntflwDoc*) stores the other two basic class categories of AQUARIUS, the

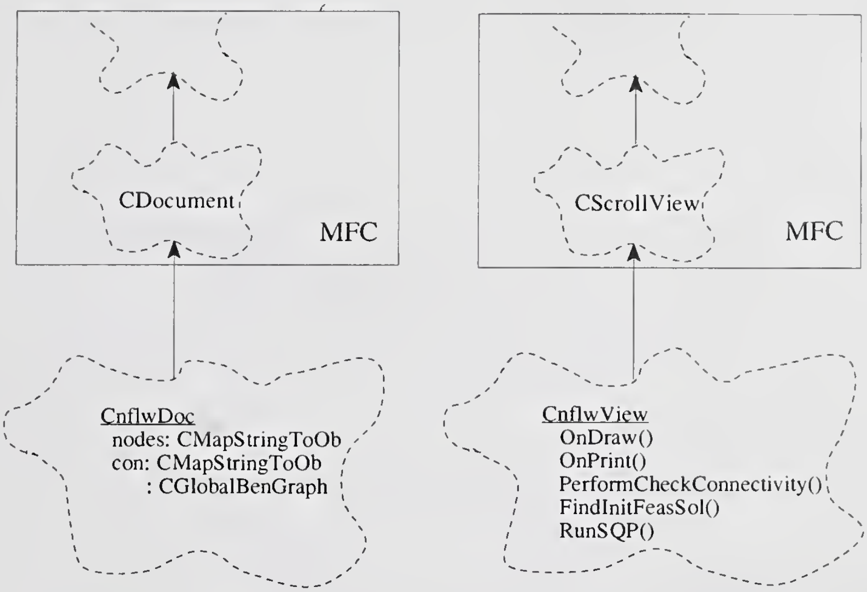


Figure 6.3 Class diagrams for the network worksheet (NWS).

Water System Components (WSC) and the Water System Links (WSL), in two different memory maps. Memory map objects are instances of the class *CMapStringToObj* defined in MFC.

In addition to the data corresponding to the individual network components, the class *CntflwDoc* also stores global data corresponding to the whole network. Global data include the selected optimization algorithm, parameters controlling the optimization, the selected period of analysis, and output data related to the optimal solution. The class *CntflwDoc* has an association relation with the class *CGlobalBenGraph*, which facilitates the update of the graphical output provided by the optimization algorithm after each sequence. This is illustrated in figure 6.4.

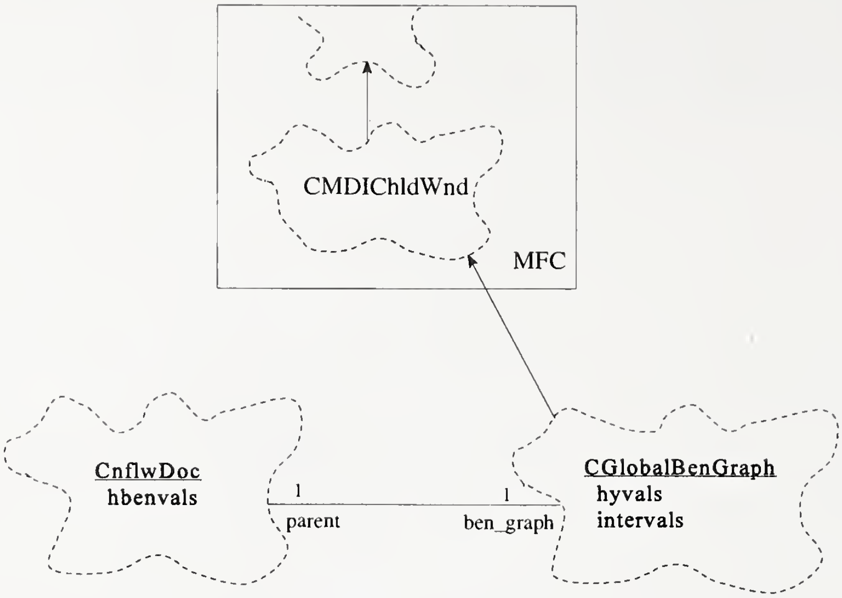


Figure 6.4 Class diagram for the network output data.

Water System Component

Water System Components (WSC) are defined by several classes, depending on the component of the flow network that requires representation. The basic class from which all WSC are derived is *CNode*. *CNode* displays runtime polymorphic behavior by storing an embedded pointer of the corresponding WSC (aggregation). The association relationship between *CNode* and the WSC is one-to-one (cardinal), which is achieved through containment by reference. *CNode* has an association relationship with the classes for the data structure. This association relationship depends on the instance of the *CNode* class (i.e., reservoir, powerplant, etc). *CNode* stores an attribute named data, which is a pointer to the data structure. Similarly, the data structure object stores an attribute named parnode, which a pointer to the *CNode* object. For example, when the user selects a reservoir from the WSC palette and drops it into the NWS, an instance of *CNode* is created and its attribute name is set equal to reservoir. At the same time, an instance of the reservoir data structure is created and embedded into the *CNode* object. Because the class for the data structure is derived from *CObject*, the reservoir object inherits the object persistence and runtime class identification properties.

As illustrated in figure 6.5, the data structure object can be an instance of any of the following classes:

- *CDataJunDiv* (junctions and diversions),
- *CDataResv* (storage capacities),
- *CDataPower* (hydroelectric plants),
- *CDataOffstream* (offstream demand areas),
- *CDataInstream* (instream demand areas), and
- *CDataWatershed* (water sources).

Figure 6.5 shows the association relation between the *CNode* Object and the respective WSC data structures. The WSC classes use other classes for user interaction, both in terms of inputs and outputs.

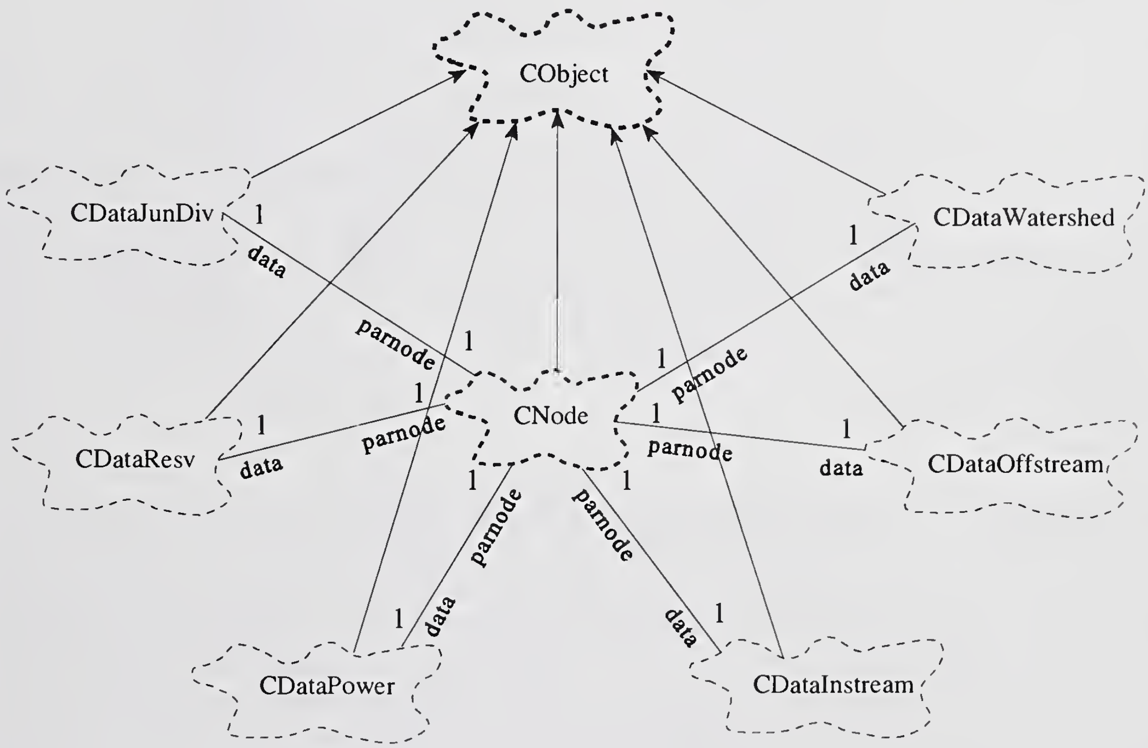


Figure 6.5 Associations between *CNode* object and the system objects.

CNode renders itself and returns an image to the view window of the NWS when the view window handles the redraw message. By design, there are eight terminals that may act as input/output connections for an object. Figure 6.6 shows the specific case of a reservoir. *CNode* stores information about which of the WSC object terminals are active or inactive and controls the terminal orientation. When the image of the WSC has to be rendered, the proper bitmap with its correct orientation is loaded. The coordinates of each terminal are defined relative to the upper left corner of the object, and are stored as an array of objects of the *CRect* class

(con_rect[8]). This allows the correct terminal to process the message when the user clicks on the WSC to make connections. Coordinate information is stored as *Cstring* objects for each terminal, which provides access to the memory map for links. Corner terminals are capable of providing only single outputs, whereas the in-between terminals are capable of multiple inputs.

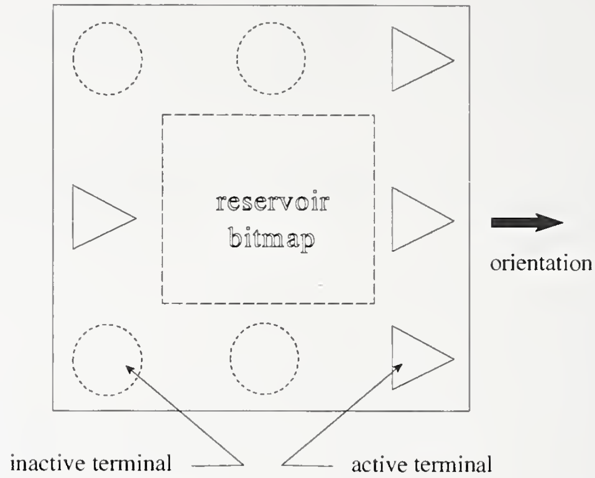


Figure 6.6 Reservoir node with input and output terminals.

In this report, we show in detail one of the classes depicted in figure 6.5, the *CDataReserv* class. This class is used to represent storage capacities. The class diagram for *CDataReserv* is shown in figure 6.7. As mentioned, the class *CDataResv* is derived from the class *CObject*. In turn, *CDataResv* has association relationships with the following classes:

- *CResvSheet*,
- *CDlgResvOut*,
- *CDlgResvTabOut*, and
- *COutGraph*.

The *CResvSheet* class, used for entering physical data for a reservoir, is derived from the *CPropertySheet* class as defined in MFC. This class has association relationships with different classes derived from *CPropertyPage*. An instance of the class *CResvSheet* is created on demand whenever the user chooses the Physical Input option from the object Tools Palette, and clicks on an instance of the reservoir class in the NWS. When the *CResvSheet* instance is created, the data for the reservoir are retrieved and displayed on a tab-dialog box. Similarly, when the user chooses the option for saving the entered data, the data are processed from the tab-dialog box and saved into the instance of the class *CDataResv*.

The *CDlgResvOut* class, used to select graphical outputs variables from the operation of a reservoir object, is derived from the *CDialog* class as defined in MFC. The object of this class is created on demand whenever the user chooses the Graphical Output option from the object tools Palette and clicks on a reservoir icon on the NWS. When a *CDlgResOut* instance is created, a dialog box is created displaying the reservoir output variables available.

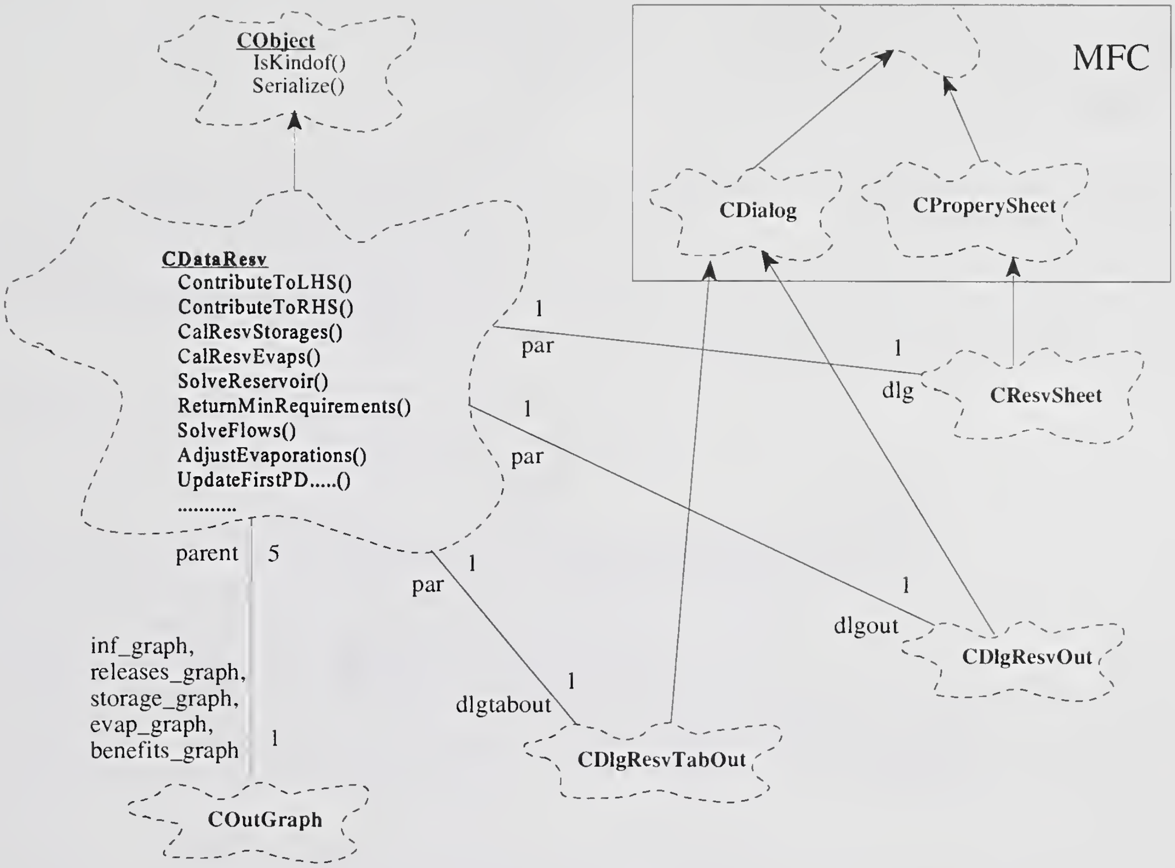


Figure 6.7 Class diagram for the *CDataResv* class.

The *CDlgResvTabOut* class, used to select output variables for the graphical/tabular folder, is also derived from the *CDialog* class as defined in MFC. When the dialog box of this class is invoked, the user can choose any of the output variables available for a reservoir and drop the variable into the graphical/tabular folder. Objects of this class are created on demand whenever the user chooses the Select Output Variable option from the objects tools palette and clicks on the corresponding reservoir icon on the NWS.

The *COutGraph* class is responsible for the display of graphical output in a window. The reservoir object has a one-to-many association relationship of cardinality 5 with this class. Consequently, the reservoir stores 5 pointers to the objects of class *COutGraph*. These objects are created to plot the following reservoir output variables: inflows, releases, storage, evaporation losses and economic revenues. Graphical output windows are created whenever the user clicks on the corresponding check box of the *CDlgResvOut* dialog box.

Water System Links

This class category is comprised of the following two subclasses:

- River Reach and
- Conveyance Structures (Canal/Pipeline).

The classes *CDataRiv* and *CDataConvey* are derived from the class *CObject* and have association relationship with the classes *CRivSheet* and *CConveySheet*, respectively (see figure 6.8). The tab-dialog boxes *CRivSheet* and *CConveySheet* are used by the analyst for entering physical input data. The dialog boxes use property pages to incorporate input data related to physical characteristics and hydraulic properties of the WSL objects.

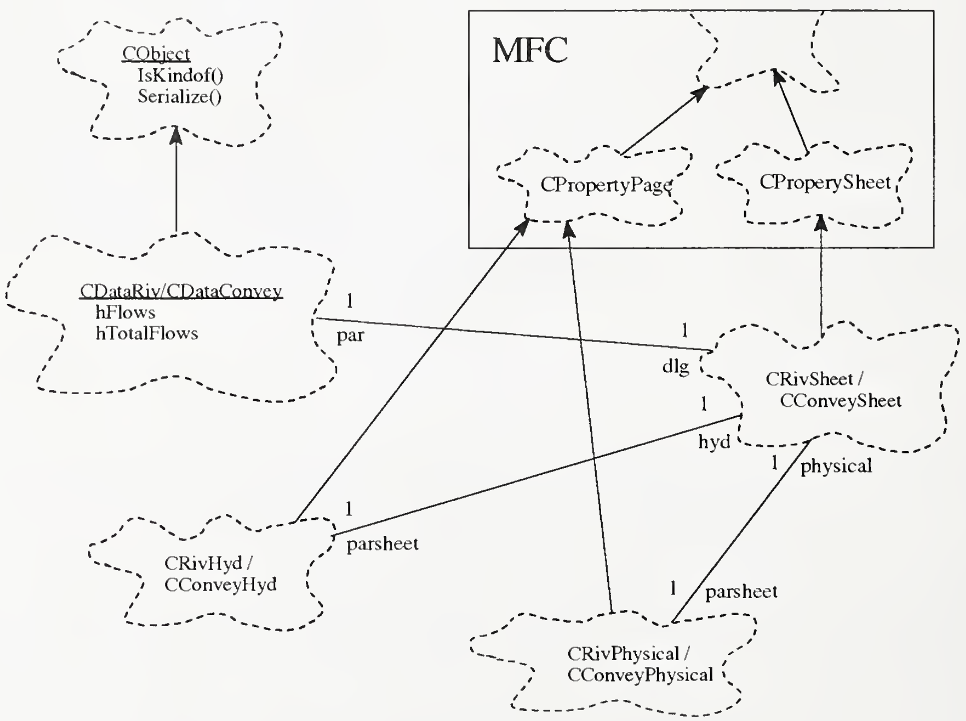


Figure 6.8 Class diagram for water systems links.

During validation of the network, discussed in Chapter 7, WSL are classified according to the place they occupy in the flow network. The classification of links is performed automatically by the model following the set of decision rules that follow:

natflow	arc used to convey water from a watershed (water source)
mandrel	(mandatory release) arc used to withdraw water from a reservoir to maintain minimum instream flow requirements immediately downstream from the reservoir

	(an FHP node has to be present)
decision	arc used to convey water into portions of the network where a water allocation decision needs to be made. Possible cases are: 1) at diversion points, offstream link of a DIV node; 2) to control direct water extraction from a reservoir serving demand areas such as HPW, IRR, M&I or IRA; and 3) to control releases from a reservoir serving other network portions.
return	arc used to carry return flows from offstream demand areas (IRR and M&I nodes) back to a water course.
spill	arc used to evacuate spillages from a reservoir.

Some arcs require a dual classification if serving more than one purpose in the network, such as:

mnd&spill	arc acting as a *mandrel* and a *spill* arc.
dec&spill	arc acting as a *decision* and a *spill* arc.

Mathematical Connectivity of System Components

Solving the water allocation problem in Chapter 5 for any user-defined network requires an automated procedure to handle the formulation mathematics. In AQUARIUS, the mathematical connectivity of the system components is derived automatically from the linkage of the objects comprising the network, which in turn reflects the direction of flow from one structure to the next (i.e., their hydraulic connectivity).

The requirements for the mathematical connectivity of the system components may vary depending on the characteristics of the optimization technique being implemented. For the optimization technique used in V96, the mathematical connectivity of a river network serves to:
1) build up the set of operational constraints (Chapter 5, Operational Restrictions) and
2) assemble the gradient vector and Hessian matrix of the second order approximation of the total objective function (Chapter 5, Solution Method)

The tasks indicated above required the development of an algorithm capable of automatically gathering information from the network about controlled and uncontrolled flows occurring upstream and downstream from a given system component. The way water sources, reservoirs, and demand zones are arranged in a river basin (i.e., network topology) determines the hydraulic and mathematical dependence among them. Controlled and uncontrolled flows occurring upstream from a given node influence the decisions at that node.

Figure 6.9 is a river system that illustrates the intricacies of mathematical connectivity. The system has two headwater reservoirs (B and C) with hydropower facilities (the powerplant connected to reservoir B is a run-of-the-river type). Releases from the powerplants annexed to the headwater reservoirs become regulated inflows to downstream reservoir A, which supplies water to most downstream demand areas including hydropower, urban supply, and a fish habitat

protection area. An irrigation demand zone and an instream recreation area are in the middle of the system.

The series of physical links (river reaches, canals/pipelines) connecting water sources, storage capacities, and demand areas are used by the model to automatically formulate the mathematical structure of the water allocation problem. First, the model identifies the decision sets that control water allocation in the flow network. Five decision sets, d_A, d_B, d_C, d_D, and d_E, are identified and randomly numbered by the model. Links conveying controlled flows are distinguished by dashed lines in figure 6.9.

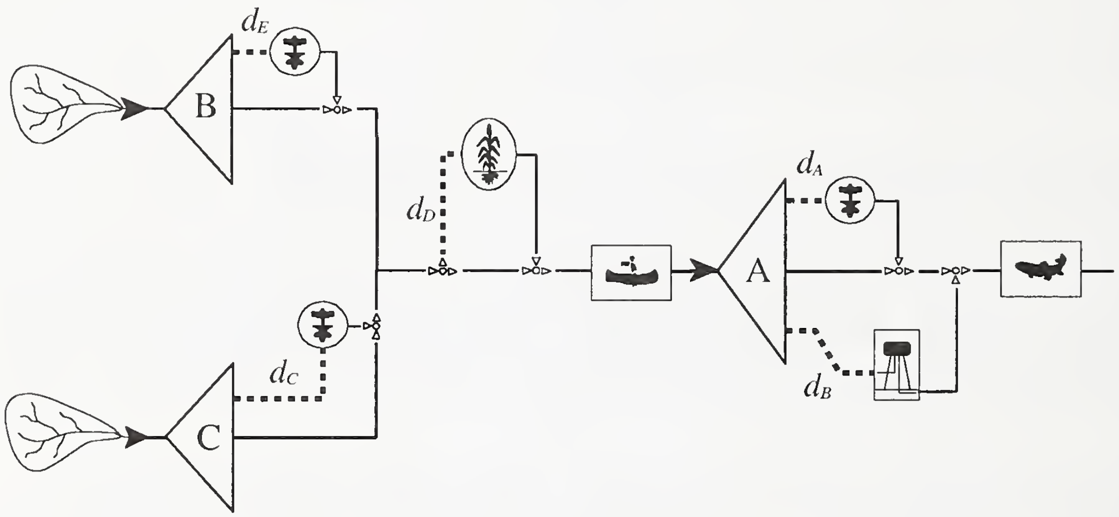


Figure 6.9 Network that demonstrates mathematical connectivity.

The model then collects information regarding all controlled and uncontrolled flows in the network using a recursive search algorithm. Some of the output generated by the search procedure is in table 6.1, which shows the coefficients of the five decision sets. For example, the first row corresponds to reservoir A, where the 1.00 values for decision sets d_C and d_E correspond to releases from the two upstream powerplants, reaching reservoir A as controlled inflows. Decision set d_D carries a coefficient equal to -0.7. The value assigned to this coefficient (less than one) indicates that 70 percent of the water diverted from the river into the irrigation area is consumptively used, with the remaining 30 percent (r = 0.3 for the irrigation area) reaching reservoir A via return flows. The remaining two coefficients, -1.00 values for decision sets d_A and d_B, represent controlled outflows from the reservoir under consideration. The values in table 6.1 can be accessed in the model using the available software menus (see Chapter 7, Exploring the Network Worksheet Screen).

Table 6.1 Table of mathematical connectivity.

d_A	d_B	d_C	d_D	d_E	Decision set
-1.00	-1.00	1.00	-0.70	1.00	Reservoir A
0.00	0.00	0.00	0.00	-1.00	Reservoir B
0.00	0.00	-1.00	0.00	0.00	Reservoir C
1.00	0.00	0.00	0.00	0.00	Hydroplant A
0.00	0.00	0.00	0.00	1.00	Hydroplant B
0.00	0.00	1.00	0.00	0.00	Hydroplant C
0.00	0.00	1.00	-1.00	1.00	Diversion
0.00	0.00	0.00	1.00	0.00	Agric. IRR
0.00	0.00	1.00	-0.70	1.00	Boating
1.00	1.00	0.00	0.00	0.00	Urban M&I
1.00	1.00	0.00	0.00	0.00	Fish habitat

Figure 6.10 shows the coefficients listed in table 6.1 plus some additional information, also gathered by the search procedure, for all nodes composing the example network in figure 6.9. This information, which is stored as part of the objects data structure, is used by the model to automatically build the set of constraint equations and compute the gradient vector and Hessian matrix. The information collected is organized in four quadrants: controlled inflows (*XI*) at the upper-left corner, uncontrolled inflows (*UI*) at the lower-left corner, controlled releases (*XR*) at the upper-right corner, and uncontrolled releases at the lower-right corner. The superscript for *UI* and *UR* indicates the originating reservoir. The reader can check the information in figure 6.10 with assistance from figure 6.9 and table 6.1.

Constraint Set Assemblage

The model is capable of attaching operational constraints to the following system components (see Chapter 5, Operational Restrictions, for more details):

- storage reservoirs (RES),
- diversion nodes (DIV),
- instream demand reaches (IRA and FHP), and
- offstream demand areas (M&I and IRR).

After the user specifies which operational constraints are included in the formulation of the water allocation problem, the NWS delegates the responsibility for building the set of restrictions to each of the components listed above, as indicated in figure 6.11.

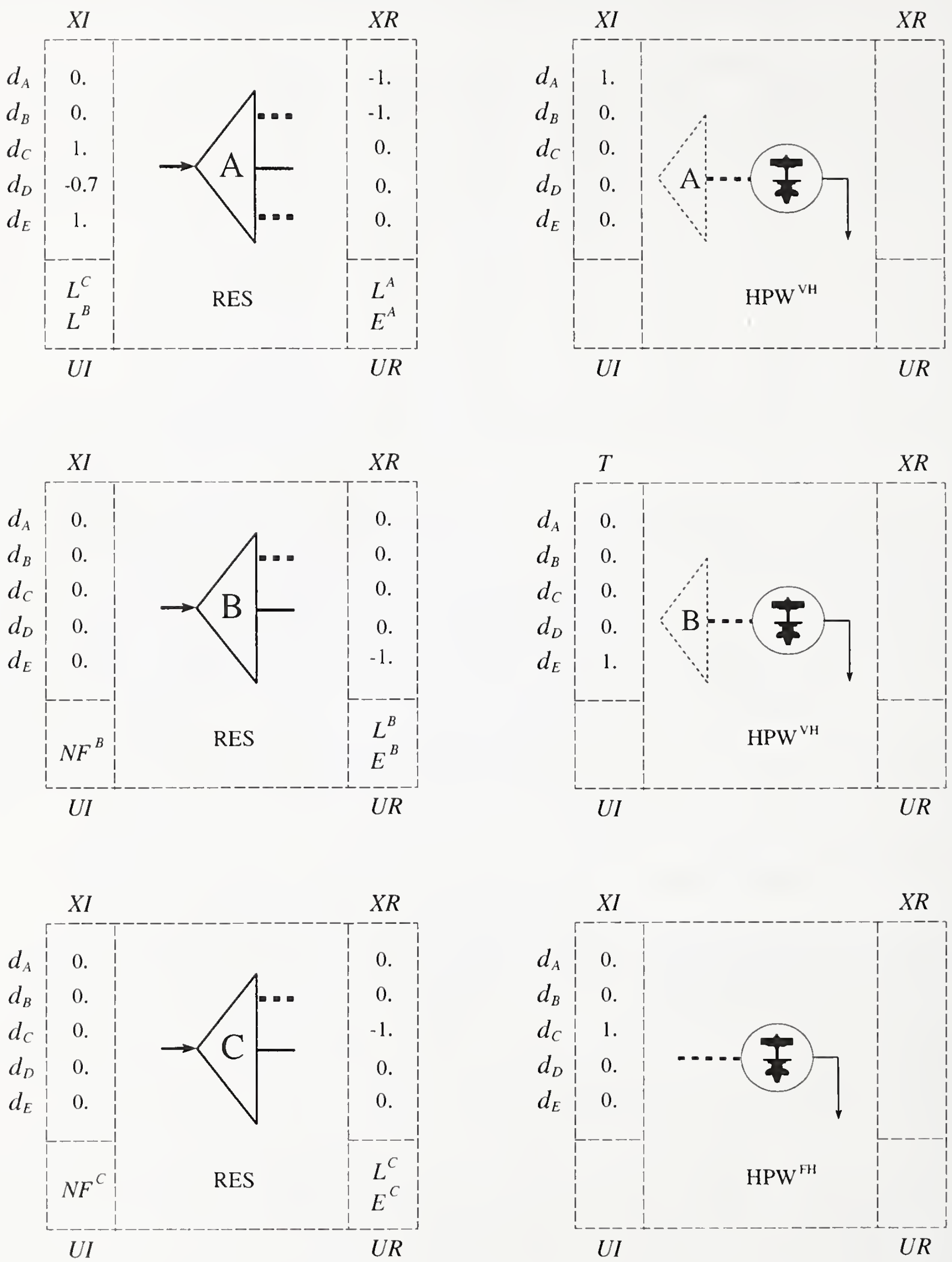
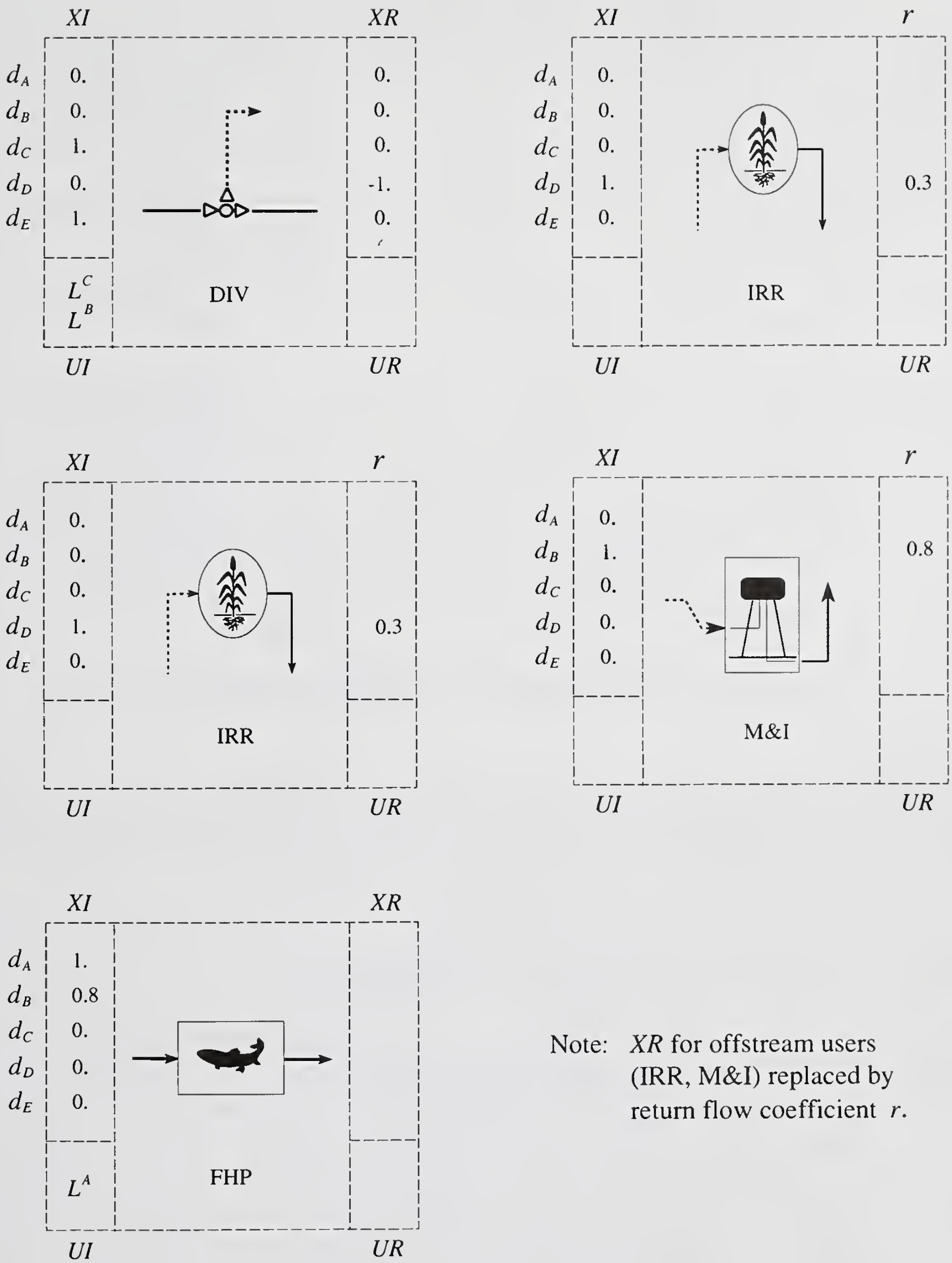


Figure 6.10 Information collected by the search algorithms.



Note: *XR* for offstream users (IRR, M&I) replaced by return flow coefficient *r*.

Figure 6.10 Information collected by the search algorithms (continued).

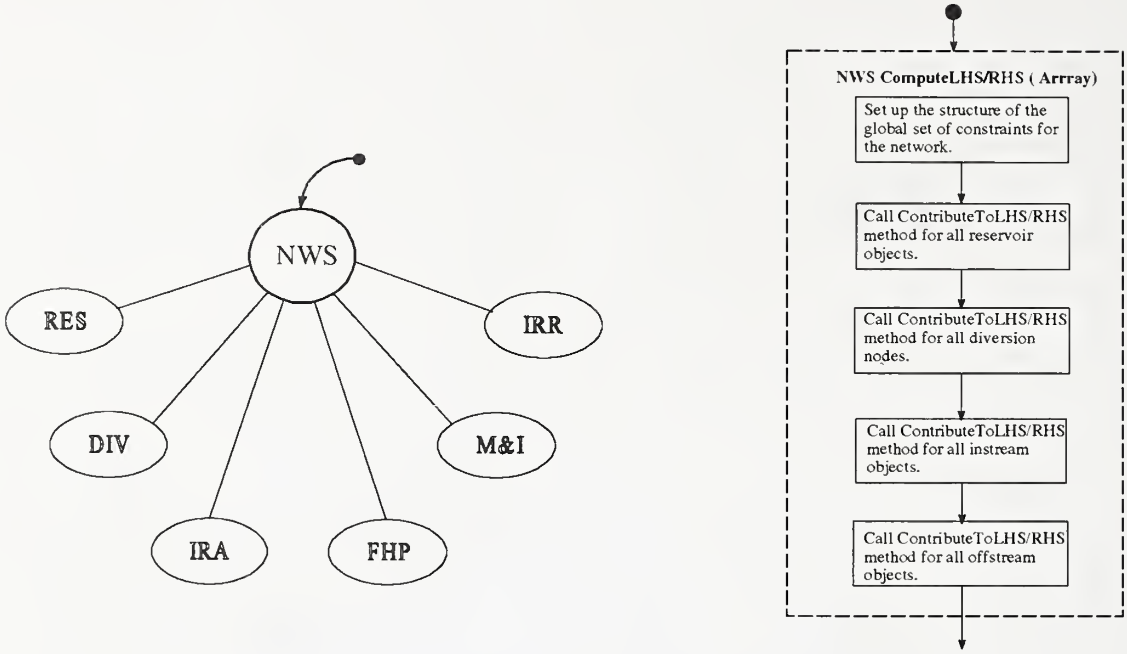


Figure 6.11 Network worksheet assemblage of constraints.

The information required by the system components for the global task of computing the right- and left-hand-sides of the equality and inequality constraints is gathered from the network, using the recursive algorithms outlined previously. Once the NWS completes the constraint set computation, the information is passed to the optimization routine.

Gradient Vector and Hessian Matrix Assemblage

How water storages and demands are arranged in a river basin determines the physical and mathematical interdependence among them. For instance, in figure 6.9, releases from the headwater powerplants become regulated inflows to the downstream reservoir, which supply water to the remaining portion of the system. The series of links connecting the various components of a flow network defines the mathematical linkage among the sets of decision variables and consequently the structure of the gradient vector ∇f and Hessian matrix H in equations (5.4) and (5.5), respectively. Diaz and Fontane (1989) demonstrated, in an earlier study on hydropower exclusively, that it is possible to identify geometrical patterns in the mathematical structure of the gradient vector and global Hessian regarding the topology of the flow network. In this study, we extended the work presented in the aforementioned publication to river basins with multiple water uses.

AQUARIUS first identifies all controlled and uncontrolled flows present in the network, then automatically builds the gradient vector and Hessian matrix. The assemblage of ∇f and H is determined based on information provided by the network search procedures and the library of partial derivatives in Appendix A. The algebraic expressions of the first and second order partial derivatives of the benefit functions were a property of each water user. For the example network under analysis, there are five basic groups of first partial derivatives ($\partial f/\partial d_A$, $\partial f/\partial d_B$, $\partial f/\partial d_C$, $\partial f/\partial d_D$, $\partial f/\partial d_E$), one for each decision set, which contain derivatives for *np* time periods. The computation of the gradient vector results from the combination of the following equations:

$$\frac{\partial f}{\partial d_A} = \text{Eq.(A.1)} \qquad \textit{for } i=1, 2, ..., np$$

$$\frac{\partial f}{\partial d_B} = \text{Eq.(A.21)} + \text{Eq.(A.2)} \qquad \textit{for } i=1, 2, ..., np$$

$$\frac{\partial f}{\partial d_C} = \text{Eq.(A.1)} + \text{Eq.(A.24)} + \text{Eq.(A.3)} \qquad \textit{for } i=1, 2, ..., np$$

$$\frac{\partial f}{\partial d_D} = \text{Eq.(A.18)} + \text{Eq.(A.24)} + \text{Eq.(A.3)} \qquad \textit{for } i=1, 2, ..., np$$

$$\frac{\partial f}{\partial d_E} = \text{Eq.(A.1)} + \text{Eq.(A.24)} + \text{Eq.(A.3)} \qquad \textit{for } i=1, 2, ..., np$$

Assemblage of the global Hessian matrix (i.e., the Hessian for the entire network) follows rules similar to those used for assembling the gradient vector, although, because of the presence of second cross-partial derivatives, it may appear more involved. Again, the mathematical connectivity of the decision sets (d_A, d_B, d_C, d_D, d_E) within the global Hessian is based on the information provided by the search algorithm and the algebraic expressions of the partial derivatives contained as part of the data structure of the objects.

Although the model requires only five decision sets to formulate the water allocation, there are six water users generating revenues in the basin: the three powerplants, the offstream demands for agriculture and urban water, and the instream demand at the recreation site. The model expresses controlled flows at the instream recreation area (IRA) as a linear combination of the upstream decision sets (see figure 6.12).

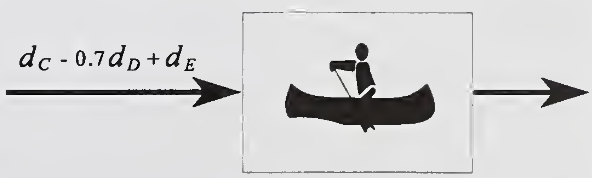


Figure 6.12 Decision variables entering the instream recreation area.

Figure 6.13 is the global Hessian matrix for the example network. The referenced numbers in the figure indicate the corresponding equations in Appendix A. Blank portions of the Hessian matrix are zero values.

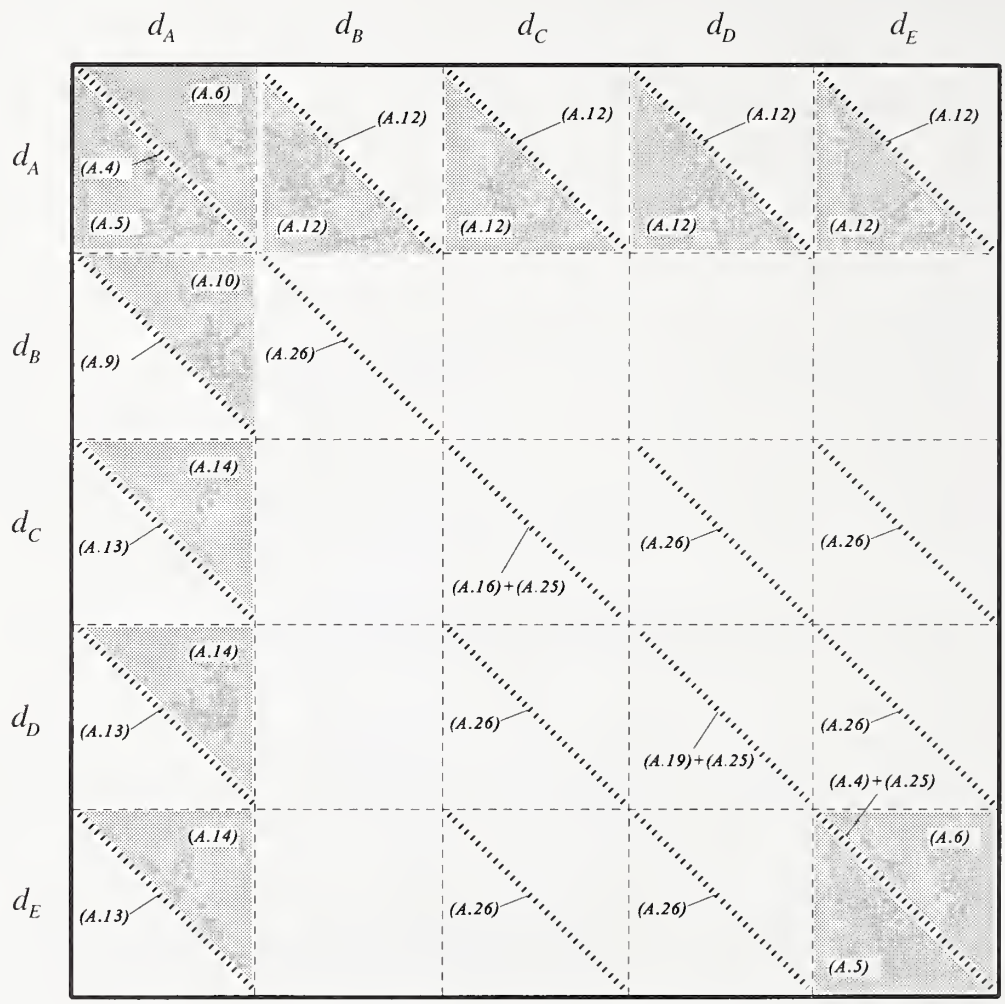


Figure 6.13 Global Hessian matrix for the example network (equations from Appendix A).

Despite the generally complex dependence among decision variables, the global H matrix is square and symmetric. Furthermore, as the model randomly selects the order of the decision sets every time the network is validated, it is possible that associations will change if the network is altered. For instance, the decision set d_A, which in the example is associated with the releases from reservoir A to the powerplant, might be associated with some other controlled release if the network was validated after an alteration was introduced. The change in the order of the decision sets would alter the arrangement of the submatrices in the Hessian matrix and possibly yield a different optimal solution of the water allocation problem due to numerical precision in the computations.

Chapter 7

How To Use AQUARIUS

This chapter describes how to use AQUARIUS including how to: 1) build a flow network using the graphical user interface, 2) enter the necessary physical and economic data, 3) solve the water allocation problem, and 4) view model outputs. We assume that the user is familiar with basic procedures for running applications in a Windows environment such as choosing commands and resizing windows.

System Requirements

AQUARIUS V96 requires an IBM-compatible personal computer with a 486 or higher processor and a 32-bit operating system (i.e., Microsoft Windows 95 or Windows NT). Enhanced performance is attained using a Pentium-based system running Microsoft's Windows NT operating system.

Exploring the Network Worksheet Screen

As described in Chapter 6, AQUARIUS was written using an object-oriented programming language. Each water system component is an object of the model, and each tool is a function that operates on an object. Although the object terminology is not repeated in this chapter, remember that the object structure of the model is what allows users to easily create a unique flow network composed of a set of water system components (WSC).

Network Worksheet (NWS) Screen

Once the AQUARIUS software is loaded in the computer, the application is launched by double-clicking the AQUARIUS icon. Initially a shortened menu selection is displayed with options for creating a new flow network or for opening an existing one by selecting the File menu. Then, the AQUARIUS screen and NWS appear (figure 7.1).

The NWS can be resized and scrolled. Several flow networks can be displayed simultaneously, although only one NWS is ever active. The title bar at the top of the NWS indicates the name of the flow network, which should have the file extension ".nwk". The menu bar at the top of the screen provides access to the various AQUARIUS commands. The two palettes at the sides of the screen contain the WSC (right side) and tools (left side) used to create the flow networks, as described later. At the bottom of the screen is the Status Line, which provides information on the currently selected object or procedure.

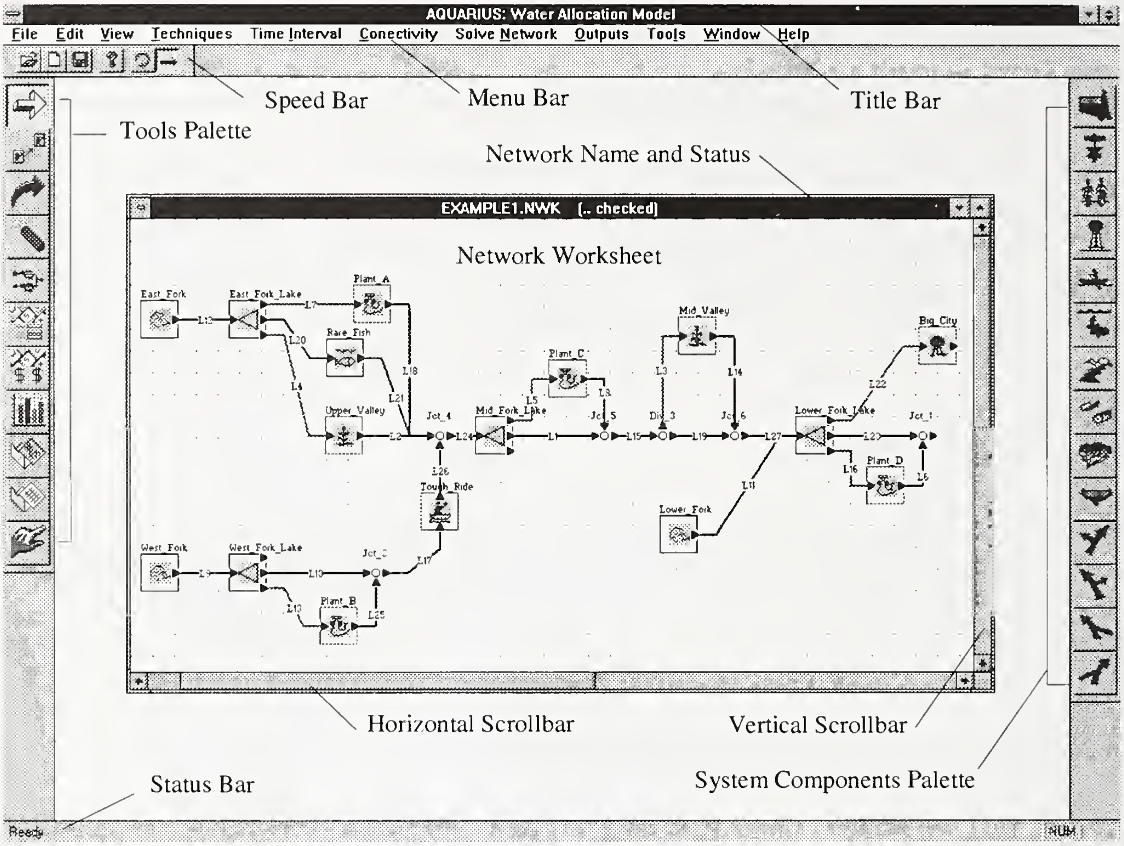


Figure 7.1 AQUARIUS network worksheet screen.

Menu Bar

Pull-down menus are arranged at the top of the screen as shown in figure 7.1. Each menu has sub-menus that allow the user to access the functions available in the model once a NWS is loaded. The complete layout of the pull-down menu bar is in figure 7.2. The command functions are explained throughout this chapter.

Water System Components (WSC) Palette

The WSC that make up a flow network are a series of nodes and links. All WSC are in a palette that is automatically displayed on the right side of the screen. The user can move the WSC palette any place on the screen including the four boundaries. The WSC palette can be removed from the screen by disabling the palette from the View menu. Choosing the WSC palette again returns it to its default position. The icons, names, and functions of the WSC are in table 7.1.

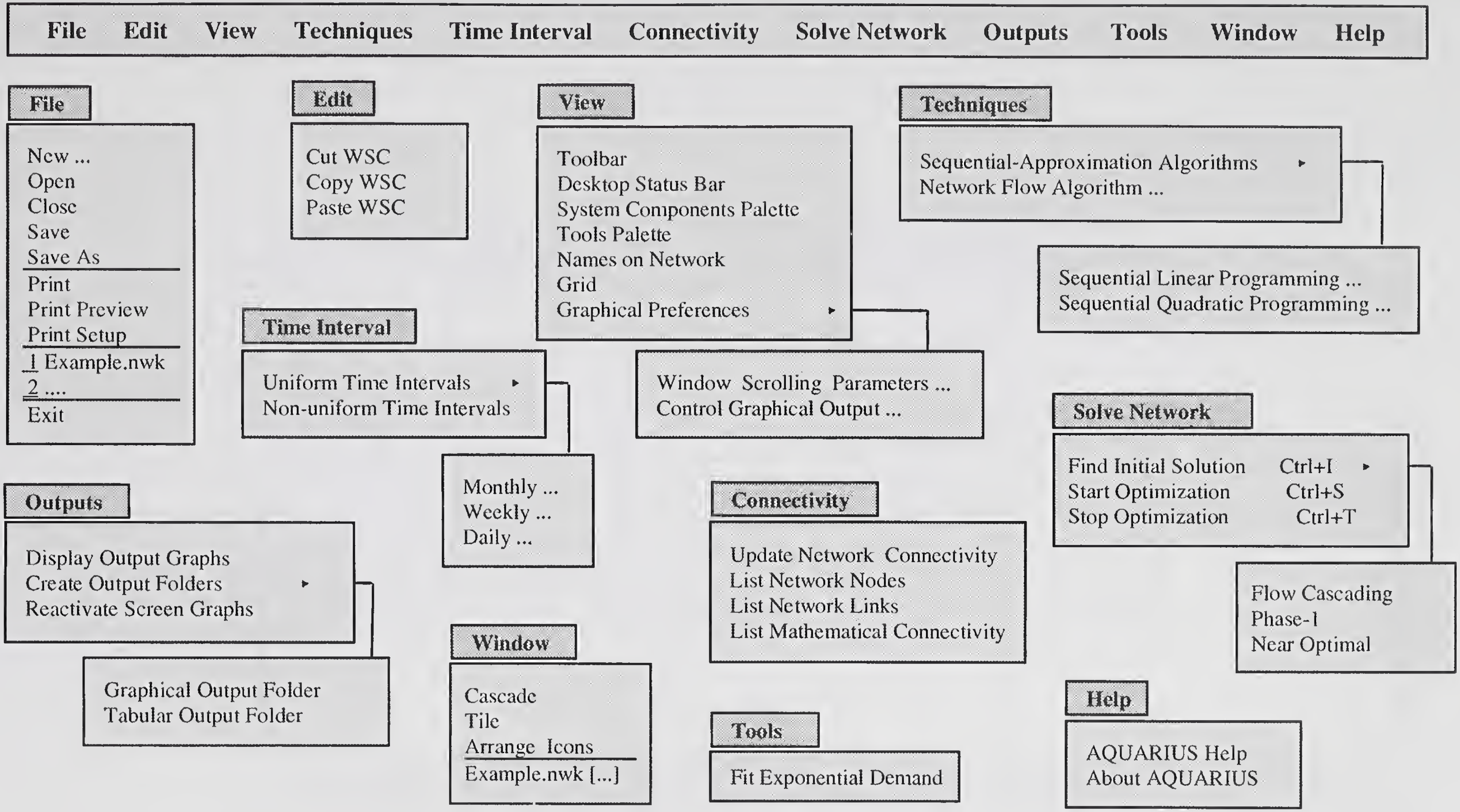
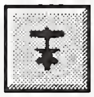
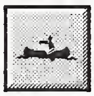
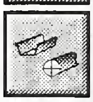
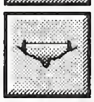
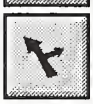
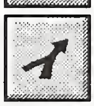


Figure 7.2 AQUARIUS pull-down menus.

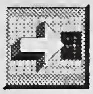
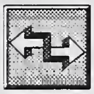
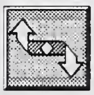
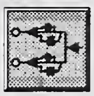
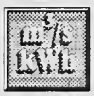
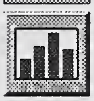
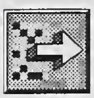
Table 7.1 The water system components palette.

Button	Name and function
	*Reservoir* .. creates a water storage capacity for flow regulation.
	*Hydropower* .. creates a powerplant for hydroelectric generation.
	*Irrigation* .. creates an offstream demand area of agricultural water.
	*Municipal and Industrial* .. creates an offstream demand area for urban and industrial water.
	*Instream Recreation* .. creates a river reach with water based recreation activities.
	*Fish Habitat Protection* .. creates a river reach designated for fish habitat protection.
	*River Reach* .. creates a natural channel reach to convey flow between system components.
	*Canal/Pipeline* .. creates a flow conveyance structure such as a canal or pipeline.
	*Natural Flow Basin* .. creates a water source area (basin) contributing uncontrolled flows.
	*Flood Control* [inactive] .. creates a river reach with structural measures for flood control purposes.
	*Left-bank Diversion* .. creates a diversion point for diverting water over the left-bank.
	*Right-bank Diversion* .. creates a diversion point for diverting water over the right-bank.
	*Left-bank Junction* .. creates an inflow point to the waterway from its left-bank.
	*Right-bank Junction* .. creates an inflow point to the waterway from its right-bank.

Tools Palette

The tools palette gives quick access to a group of functions that operate the WSC. All tools are in a palette that is automatically displayed on the left side of the screen. The user can move the tools palette to any of the four sides of the screen. The tools palette can be removed from the screen by disabling tools palette from the View menu. Choosing tools palette again returns it to its default position. The icons, names, and functions of the tools are in table 7.2.

Table 7.2 The tools palette.

Button	Name and function
	*Select* .. selects one or more system components from a flow network.
	*Move* .. moves a system component within the network worksheet.
	*Rotate* .. rotates the direction of flow in a system component.
	*Delete* .. deletes a system component from the network worksheet.
	*Physical Connectivity* .. displays categories of in/outflows from a system component.
	*Physical Input* .. accesses the physical data structure of a system component.
	*Economic Input* .. accesses the economic data structure of a system component.
	*Output Graph* .. accesses the list of output variables of a system component.
	*Graphical Folder* .. opens a folder that creates a list of variables for output in graphical format.
	*Tabular Folder* .. opens a folder that creates a list of variables for output in tabular format (ASCII file).
	*Select Output Variables* .. moves the output variables into the *Graphical /Tabular Folder.*

Creating a Flow Network

The user interacts with the model through a graphical user interface to create a flow network. The graphical user interface consists of the four elements of the model introduced earlier: 1) the network worksheet (NWS), 2) the menus, 2) the water system components (WSC) palette, and 4) the tools palette.

In the NWS, each system component corresponds to a graphical node or link of the flow network. The components are represented by icons in the WSC palette, based on a pictorial representation of the component's function. By selecting (left clicking) an icon from the WSC palette and then left clicking again in the NWS, the model creates an instance of the component. In this manner, all the necessary components that simulate the topology of the actual river system are created.

To assemble a flow network, place two or more nodes (water sources, reservoirs, powerplants, demand areas, diversions, or junctions) in the NWS. Next, link the nodes using natural river reaches and canal/pipelines, also selected from the WSC palette. Once a link is selected, place it in the NWS by clicking first on an outgoing terminal of the upstream node and then on an incoming terminal of the downstream node, always in that order. Additional nodes and links can be added to a network, but links can only be placed between existing nodes.

There is no limit to the number of components that can be added to a network, but doing so increases the mathematical dimension of the water allocation problem and the computation time needed to find the optimal solution (see reference to computation times at the end of Chapter 8). In placing links, the model prevents absurd connections such as routing water released from a reservoir back as an inflow to the same reservoir. Moreover, the model will automatically disallow connections that are hydraulically unsound or not allowed by the present version of the model.

Components can be moved anywhere in the NWS by clicking the Move button from the tools palette, clicking on the component, and dragging and dropping it to its new position. A component can be removed from the NWS by clicking the Delete button of the tools palette and then clicking on the component in the NWS. Delete removes the icon from the screen and eliminates that instance of the component from the model.

The creation and alteration of flow networks is expedited by copying and inserting portions of an existing network onto the same or a new NWS. To do this, click the Select button of the tools palette and select the part of the existing network to copy to the Windows clipboard. Select the desired part by dragging a rubber-band box around it. The starting point is the upper-left corner of the box, which is anchored by left-clicking at the location. The box is enlarged by dragging the pointer to its final position. The screen colors are reversed within the box (figure 7.3). To copy the information in the box into the Windows clipboard, choose Copy WSC under the Edit menu. To place the information from the clipboard to a NWS, select the command Paste WSC from the Edit menu and click at the desired location within the NWS. The Paste WSC command is available whenever the Windows clipboard has information that can be brought into a NWS.

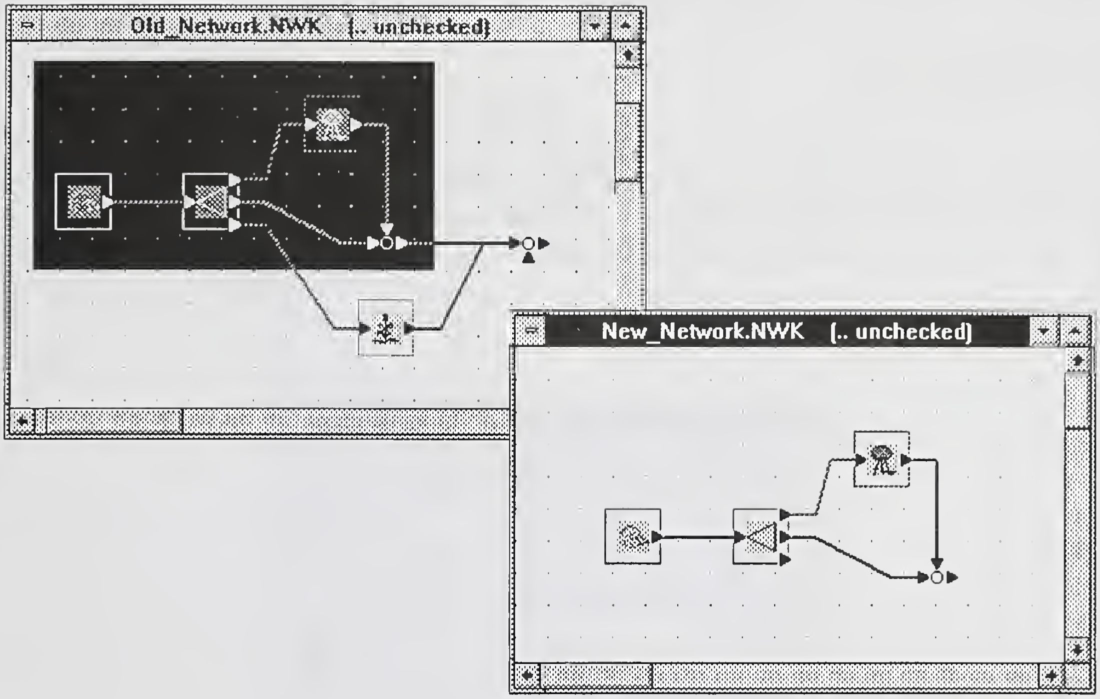


Figure 7.3 Copy and Paste of network components.

Because Copy/Paste creates new instances in the NWS of the components stored on the clipboard and duplicates their data structure, we call the duplicated objects clones. After pasting new components in a NWS, amend the data associated with the new components if they require different data from that brought in with the pasting operation. Information on the clipboard can be placed into numerous worksheet locations as long as it remains in the clipboard. Copy/Paste also provides the opportunity to build a library of individual components (e.g., reservoirs, irrigation areas) or small networks, which can be easily incorporated onto a NWS when necessary.

Entering Input Data

The model uses physical and economic groups of input data. The physical data consist largely of the dimensions and operational characteristics of the system components such as maximum reservoir storage capacity, powerplant efficiency, and the return flow coefficient from an irrigation zone. The economic data consist mainly of the demand functions of the various water uses competing for water within the river system. The input data entered for any system component is a property of the object, even when stored on disk for future use. When the

network is reloaded from the disk to the NWS, all data saved from the previous session for each component are retrieved in exactly the same form.

Physical Input Data

Physical data are entered into the model using a series of dialog boxes specifically designed for each system component. The data entry procedure is selected by clicking the Physical Input button from the Tools palette and clicking the component of interest. A data entry dialog box opens with multiple tabs that group the input data by categories (figure 7.4). Table 7.3 lists the data required for each system component.

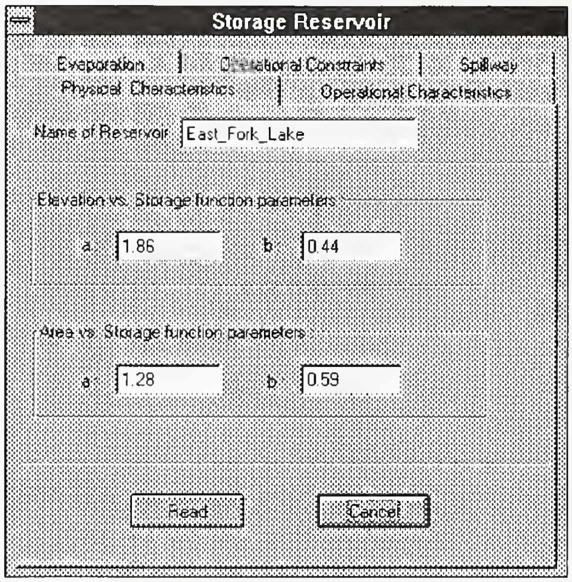


Figure 7.4 Physical data input dialog box for a reservoir.

Table 7.3 Physical inputs for the system components.

System component	Input data [units]	Reference
Reservoir	Physical Characteristics	
	Name of Reservoir [alphanumeric]	
	Elevation vs. Storage Function, Parameters : c_1, d_1	Eq.(3.4a)
	Area vs. Storage Function : c_2, d_2	Eq.(3.4b)
	Evaporation	
	Seasonal evaporation rates [mm]	
	Operational Characteristics	Fig. 3.2
	Initial storage [Mcm]	
	Minimum storage [Mcm]	
	Maximum storage [Mcm]	
	Final storage [Mcm]	

	Operational Constraints (check-box)	Sec. 5.5
	☒ Minimum storage	
	☒ Maximum storage	
	☒ Final storage	
	Spillway	Eq.(3.6)
	Spillway length [m]	
	Spillway crest elevation [m]	
	Spillway discharge coefficient	
	Parameters of the Discharge-Elevation Function:	
	Parameters of the Capacity-Surcharge Function:	
Hydropower	Physical Characteristics	
	Name of hydropower plant [alphanumeric]	
	Installed capacity [MW]	
	Design Discharge [m^3/s]	
	Turbine-generator efficiency [-]	Eq.(3.8)
	Energy Rate vs. Storage function, Parameters: a_1 , b_1	Eq.(4.1)
	Tailwater Elevation vs. Storage function.	
	Firm Energy:	
	Seasonal Pattern [GWh]	
	Maintenance Schedule	
	Index of availability of the hydropower units [-]	
	Maximum Release	
	Seasonal values [Mcm]	
	Minimum Release	
	Seasonal values [Mcm]	
	Operational Constraints (check-box)	
	☒ Minimum release	
	☒ Maximum release	
	☐ Firm energy	
Irrigation	Physical Characteristics	
	Name of irrigation area [alphanumeric]	
	Coefficient of return flow [-]	Fig.4.5
	Maximum Flow	
	Seasonal values [Mcm]	
	Minimum Flow	
	Seasonal values [Mcm]	
	Operational Constraints (check-box)	
	☐ Minimum flow	
	☐ Maximum flow	
	☐ Seasonal Pattern	
	☐ Annual Firm Water	
Municipal & Industrial	Physical Characteristics	
	Name of municipal and industrial area [alphanumeric]	
	Coefficient of return flow	Fig.4.6
	Maximum Flow	
	Seasonal values [Mcm]	
	Minimum Flow	
	Seasonal values [Mcm]	
	Operational Constraints (check-box)	

- □ Minimum flow
- □ Maximum flow
- □ Seasonal Pattern
- □ Annual Firm Water

Instream Recreation	Physical Characteristics Name of instream recreational area [alphanumeric] Maximum Flow Seasonal values [Mcm] Minimum Flow Seasonal values [Mcm] Operational Constraints (check-box) □ Minimum flow □ Maximum flow
Instream Flow Protection	Physical Characteristics Name of conservation area [alphanumeric] Maximum Flow Seasonal values [Mcm] Minimum Flow Seasonal values [Mcm] Operational Constraints (check-box) □ Minimum flow □ Maximum flow
Flood Control	(not implemented)
Natural Flow Basin	Physical Characteristics Name of river/basin [alphanumeric] Input file Name [Path....]
River Reach	Physical Characteristics Name of river reach [alphanumeric] Connectivity, From: __ To: __ Description [alphanumeric] Length [m] Hydraulic Characteristics Maximum flow capacity [m^3/s] Flow velocity [m/s] Channel losses [l/km] Flow routing (check- box) Routing Parameters: k, x
Canal/Pipeline	Physical Characteristics Name of river reach [alphanumeric] Description [alphanumeric] Length [m] Connectivity, From: __ To: __ Hydraulic Characteristics Maximum flow capacity [m^3/s] Flow velocity [m/s] Canal/pipe losses [l/km]

Diversion	Physical Characteristics □ Node water-balance (check-box)
Junction	(No input data required)

Note: an option is enabled when a check mark appears in the check box; disabled when the check box is empty.
Variables in shaded font are unavailable in version V96.

Economic Input Data

The economic data are the parameters necessary to specify the demand curves (i.e., the marginal prices that users are willing to pay) for water during each season of the optimization horizon. The data entry procedure is selected by clicking the Economic Input button from the tools palette and clicking the object of interest. A data entry dialog box opens for user interaction (figure 7.5). Table 7.4 lists the economic data required for each competing water use.

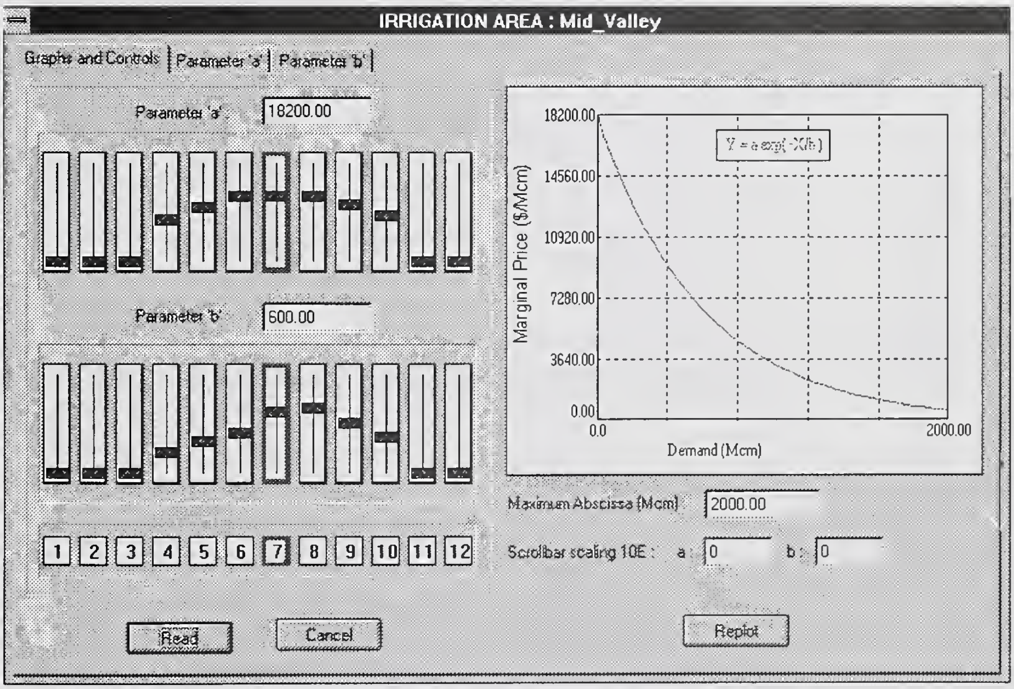


Figure 7.5 Economic data input dialog box for an irrigation area.

Table 7.4 Economic inputs for the system components.

System Component	Input Data [units]	Reference
Reservoir	Lake Recreation Benefits	
	Demand Curve, Parameters	
Hydropower	Energy Generation Benefits	
	Average energy price: P [$/MWh]	Eq.(4.3)
	Seasonal energy prices: P_s [$/MWh]	Eq.(4.3)
	Energy demand function; Parameters: a_2, b_2	Eq.(4.3)
Irrigation	Agricultural Benefits	
	Seasonal demand function; Parameters: a_3, b_3	Eq.(4.15)
Municipal & Industrial	Urban and Industrial Water Supply Benefits	
	Seasonal demand function; Parameters: a_4, b_4	Eq.(4.20)
Instream Recreation	Instream Recreation Benefits	
	Seasonal demand function; Parameters: a_5, b_5	Eq.(4.25)
System (whole network)	Present Worth Analysis	
	Number of discounting periods	
	Discounting rate [%]	
	Price escalation rate [%]	
	Value of the objective function expressed in:	
	☐ Thousands [$]	
	☒ Millions [$]	

Updating Network Connectivity

Once the flow network is assembled, the model conducts a series of validation steps before proceeding to solve the water allocation problem. These procedures are launched by choosing the command Update Network Connectivity from the Connectivity menu. Among the various validation steps, the model checks that:

- all WSC are named;
- the WSC that compete for water have marginal prices greater than zero;
- water uses such as IRA, FHP, and FCA are instream users;
- reservoirs have a waterway available for releasing uncontrolled spills; and
- component connections are hydraulically sound.

Warning messages are displayed for violations of preestablished network connectivity rules. If all steps are successfully completed, the network status changes from *..unchecked* to *..checked* on the title bar of the NWS.

System components competing for water are framed by a red line on the screen. Similarly, links conveying controlled flows (i.e., links leaving a node where a decision is made) are turned red. The number of sets of decision variables is equal to the number of red links. The WSC frame and the links turn red after updating network connectivity.

An additional operation performed during this stage is defining the hydraulic connectivity of the network objects. This information is used by the model to generate the mathematical connectivity of the system components based on the requirements of the optimization technique selected. Details on the mathematical connectivity of the system elements are in Chapter 6.

If Names on Network is chosen from the View menu, next to each component in the NWS the model displays the name assigned to the component by the user and the number assigned by the model to links (i.e., river reaches and conveyance structures) and to diversion and junction nodes. The user can also look at and print a table summarizing the network nodes (figure 7.6 top) and another one summarizing network links (figure 7.6 bottom). These tables are accessed by the commands List Network Nodes and List Network Links in the Connectivity menu.

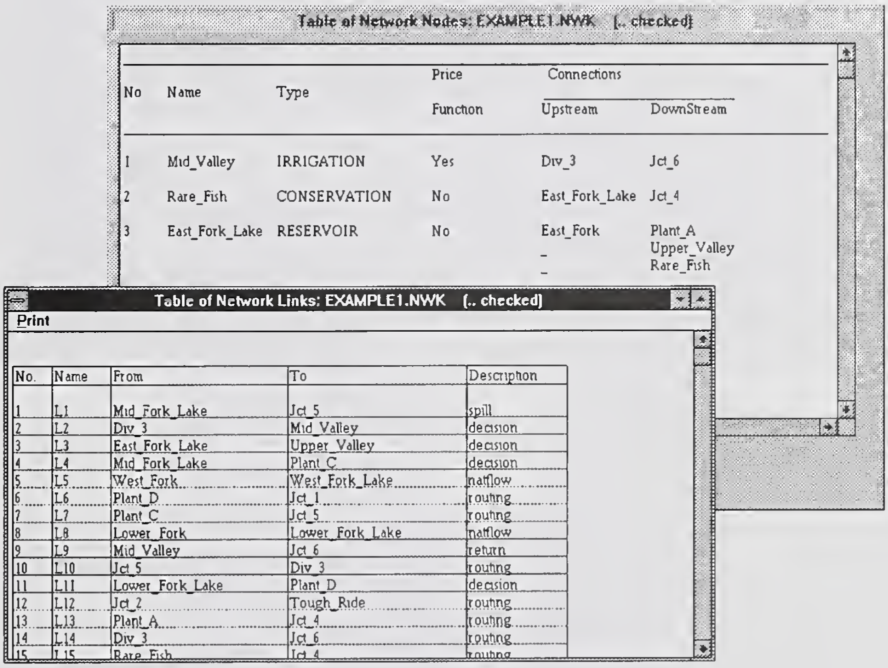


Table of Network Nodes: EXAMPLE1.NWK [.. checked]

No	Name	Type	Price Function	Connections Upstream	Connections DownStream
1	Mid_Valley	IRRIGATION	Yes	Div_3	Jct_6
2	Rare_Fish	CONSERVATION	No	East_Fork_Lake	Jct_4
3	East_Fork_Lake	RESERVOIR	No	East_Fork	Plant_A
				_	Upper_Valley
				_	Rare_Fish

Table of Network Links: EXAMPLE1.NWK (.. checked)

Print

No.	Name	From	To	Description
1	L1	Mid_Fork_Lake	Jct_5	spill
2	L2	Div_3	Mid_Valley	decision
3	L3	East_Fork_Lake	Upper_Valley	decision
4	L4	Mid_Fork_Lake	Plant_C	decision
5	L5	West_Fork	West_Fork_Lake	natflow
6	L6	Plant_D	Jct_1	routing
7	L7	Plant_C	Jct_5	routing
8	L8	Lower_Fork	Lower_Fork_Lake	natflow
9	L9	Mid_Valley	Jct_6	return
10	L10	Jct_5	Div_3	routing
11	L11	Lower_Fork_Lake	Plant_D	decision
12	L12	Jct_2	Tough_Ride	routing
13	L13	Plant_A	Jct_4	routing
14	L14	Div_3	Jct_6	routing
15	L15	Rare_Fish	Jct_4	routing

Figure 7.6 Table of network nodes and connectivity links.

Selecting the Optimization Technique

AQUARIUS V96 includes two sequential optimization techniques for solving the water allocation problem: 1) Sequential Linear Programming (SLP) and 2) Sequential Quadratic Programming (SQP). Details of the algorithms are in Chapter 5. The algorithms are selected by choosing the commands Sequential Linear Programming or Sequential Quadratic Programming from the Techniques menu. These commands open a dialog box containing a series of parameters that control the sequential process and the accuracy of the computations (figure 7.7).

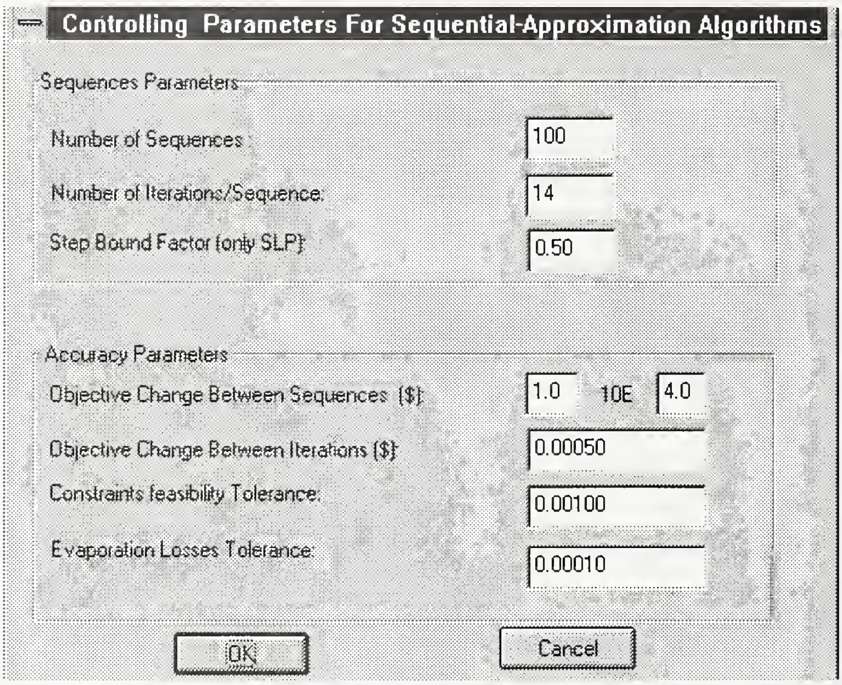


Figure 7.7 Control parameters dialog box.

The following three parameters are specified by the user in the dialog box to control the sequential approximation algorithm: 1) The Number of Sequences. This defines the maximum number of LP/QP sequential problems to be solved (Figure 5.1). The default for this parameter is 100, which is a reasonable number for the example network in Chapter 8. 2) The Number of Iterations per Sequence. This parameter is intrinsic to the general differential algorithm used by QPTHOR to solve the successive LP/QP problem. A reasonable selection for the example network in Chapter 8 is 10 to 15. 3) The Step Bound Factor. This parameter is intrinsic to the SLP algorithm (see the discussion at the end of the Solution Method section of Chapter 5). It can adopt values between 0.25 and 0.75 and has a default of 0.5

The final solution to the water allocation problem found by the model can be sensitive to the values assigned to the controlling parameters listed above. The proper values for the parameters will depend on the topology of the network and the length of the optimization horizon. Given the

difficulty of preestablishing the proper value for these parameters, the user should experiment with different values to find the most appropriate numbers for the problem.

Four accuracy parameters are specified by the user to control the optimization calculations. The purpose of each parameter is in the dialog box (figure 7.7). The first parameter, Objective Change between Sequences, is the main stop criterion. Five consecutive solutions of the problem with an objective function change less than the specified dollar amount will cause the application to stop. To set the other three accuracy parameters, the user should begin running the model with the default values and then modify the parameters as a better understanding of the intricacies of the network is gained.

Specifying the Simulation Time Interval

The time interval for the analysis is specified by selecting the Uniform Time Intervals from the Time Interval menu. This opens a submenu for selecting the periodic time interval: *Monthly...*, *Weekly...*, *Daily....* Once the periodic time interval is selected, a window opens to enter the four items listed in the dialog box in figure 7.8. Although the model was conceived to operate at any of the uniform time steps offered in the menu (monthly, weekly, daily), present limitations in the graphical user interfaces allow for only monthly time intervals.

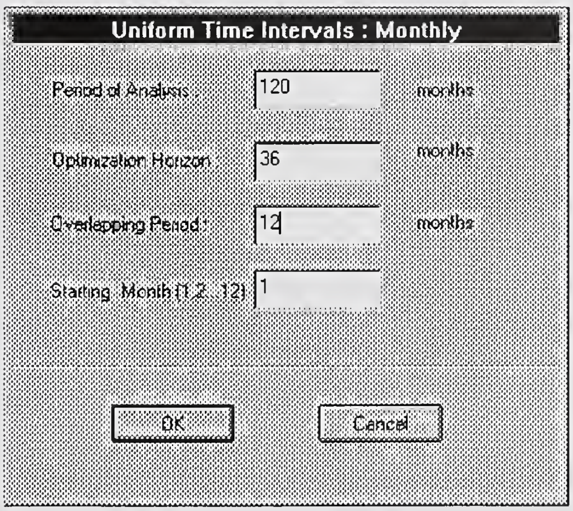


Figure 7.8 Period of analysis and optimization horizon dialog box.

The first entry in figure 7.8, Period of Analysis, specifies the length, in number of periods, of the whole segment of time for which the model will determine the system operation. Inflow data should be provided for at least the same segment of time. The next entry, Optimization Horizon, also in number of time periods, specifies how far into future the model should look to build the

optimal operational policies. The optimization horizon defines the actual size of the optimization problem(s) being solved.

If the Optimization Horizon is set equal to the Period of Analysis, the quasi-continuous optimization scheme is disabled (see Chapter 5, Time Intervals of Analysis); hence, the optimal operational policies identified by the model will extend for the full number of periods indicated by the optimization horizon. However, when the period of analysis is longer than the optimization horizon, which enables the quasi-continuous optimization procedure, the user must also specify the number of Overlapping Periods. The model allows the user to prescribe only 6, 12, or multiples of 12 overlapping periods for monthly simulation.

The last input in the dialog box in figure 7.8, Starting Month, indicates the period within the annual cycle for the start of the system operation. For instance, 1 for January, 2 for February, . . 12 for December (for a monthly time step). In this manner, time series data, such as inflows, evaporation rates, etc., are properly considered.

Solving the Water Allocation Problem

Once the network is assembled, including input of all the physical and economic data, and the network connectivity is verified, AQUARIUS can find the optimal water allocation. This may occur in one or two steps. First, an initial feasible solution (IFS) to the water allocation problem must be found. The IFS that the model finds will generally be far from the optimal solution (see Chapter 5, Search for a Feasible Solution), but serves as the starting point for the sequential optimization procedure that follows. The IFS is obtained by selecting Find Initial Solution from the Connectivity menu and selecting one of the algorithms. Find Initial Solution acts only on a network in the *..checked* condition (see figure 7.1). The user is prompted to enter a file name indicating where on disk the decision values and state of the system information will be saved. The graphical output capabilities of the model, introduced next, can be used to render the state of the system that corresponds to the initial feasible solution found by the model.

Alternatively, the user may search directly for the optimal solution, bypassing displaying and saving an IFS; the search for an IFS still occurs but is invisible to the user. The final step, actual optimization, consists of running an optimization algorithm to find the optimal solution of the water allocation problem. This is accomplished by selecting the command Start Optimization from the Solve Network menu. As explained in Chapter 5, sequential algorithms go through a succession of linear or quadratic approximations of the nonlinear global objective function until the optimal solution is reached. The maximum number of LP or QP sequences allowed is indicated in the corresponding dialog box (figure 7.7). As the solutions of the LP/QP problems progress, the status bar at the bottom of the screen displays the sequence number. Once the optimal solution is found, or the maximum number of sequences is reached, the optimization stops (indicated in the status bar). The sequential optimization process can be interrupted by selecting Stop Optimization from the Solve Network menu. The user can track the sequential

changes in system state and decision variables during the optimization by using the graphical output capabilities presented next.

Graphical and Tabular Output

Flow networks generally contain many state, decision, and economic variables that an analyst should consider. AQUARIUS facilitatés the interpretation and analysis of all information through readily accessible graphical and tabular output display formats. The user can display output variables on the screen by selecting Display Output Graphs from the Outputs Menu or by pressing the corresponding icon in the tools palette and clicking on the component of interest in the NWS. This opens a dialog box that lists all physical and economic output variables available for that component. Figure 7.9 shows a dialog box for a reservoir. A graph window is created when a check mark is inserted (enabled) in the corresponding check box and is deleted (disabled) when the check box is empty.

If instead of clicking on a WSC in the NWS, the user clicks any place in the worksheet except the area occupied by a system component, a graph window opens showing the evolution of the total-objective function value as the sequential algorithm (SLP/SQP) completes optimization. This is possible because the flow network as a whole is also an object (Chapter 6).

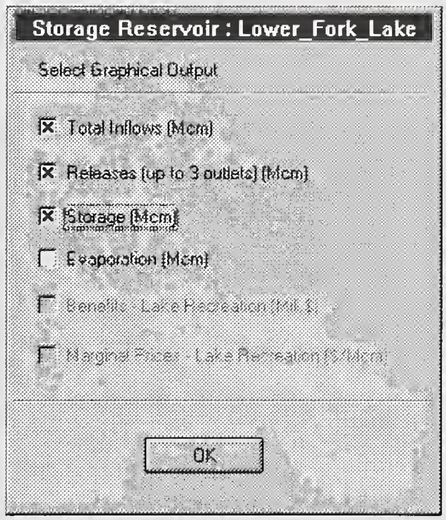


Figure 7.9 Selecting graphical display output variables for a reservoir.

Single-variable graph windows allow the user to visualize the sequential changes in the state, decision, and economic output variables as the system reaches its optimal state. Visualizing sequential changes is only possible if the output graph windows were opened either before launching optimization or during the optimization (e.g., immediately after the IFS was found).

Several graph windows can be open simultaneously, the only limitation is the number of windows visible on the screen at one time. As they are opened, the graph windows appear on the screen in a cascade format. The user may choose Tile from the Window menu to display all graph windows at once. The model maintains a register of all graph windows opened during a working session. The same graph windows can be reactivated any time by choosing Reactivate Output Graphs from the Outputs menu. The continuous refreshing of the graphical output windows after each sequence of optimization increases the execution time of the model.

Table 7.5 presents the output variables available for graphical display, listed by type of water system component.

Table 7.5 Output variables for the water system components.

Reservoir
- □ Total Inflows [Mcm]
- □ Releases (up to 3 outlets) [Mcm]
- □ Storage [Mcm]
- □ Evaporation [Mcm]
- □ Benefits - Lake Recreation [Mill.$]
- □ Marginal Prices - Lake recreation [$/Mcm]

Hydropower
- □ Powerplant Flows [Mcm]
- □ Hydropower Generation [GWh]
- □ Benefits - Hydrogeneration [Mill.$]
- □ Marginal Prices - Hydrogeneration [$/Mcm]

Irrigation
- □ Allocated Flows [Mcm]
- □ Return Flows [Mcm]
- □ Benefits - Agricultural Use [Mill.$]
- □ Marginal Prices - Agricultural Use [$/Mcm]

Municipal & Industrial
- □ Allocated Flows [Mcm]
- □ Return Flows [Mcm]
- □ Benefits - Urban Use [Mill.$]
- □ Marginal Prices - Urban Use [$/Mcm]

Instream Recreation
- □ Instream Flows [Mcm]
- □ Benefits - Instream Use [Mill.$]
- □ Marginal Prices - Instream Use [$/Mcm]

Instream Flow Protection
- □ Instream Flows [Mcm]
- □ Ecological Index

Natural Flow Basin
- □ Natural Flows [Mcm]

Network
- □ Global Objective Function [Mill.$]
- □ Periodic Series of Benefits [Mill.$]

In addition to single-variable graphs, the user can create graphical and tabular outputs containing one or more physical or economic variables. This is useful for displaying, in the same graph,

output variables of the same class generated at different locations in a flow network. For instance, the user may want to compare energy generated at different powerplants, instream versus offstream water allocations, or storage trajectories for all reservoirs. Similarly, this option can be used to generate tabular outputs (in ASCII format) that can be printed or imported to a spreadsheet. This is done by selecting Create Output Folders from the Outputs menu. This opens a submenu from which to choose either a Graphical or a Tabular Output Folder. Folders can also be opened from the tools palette. Both choices open a new window, the folder, that will list all output variables. Once the folder window is visible, the procedure for selecting the variables starts by clicking the Select Output Variable button of the tools palette and clicking the component of interest in the NWS. This opens another window that lists all output variables for that specific component (table 7.5 and figure 7.10). Select the output variables of interest by dragging each one from the Select Output Variable window and dropping it into the Output Folder window (figure 7.10). Once all variables are selected, create an output product (i.e., graph or file) by clicking on the Create Graph (File) button.

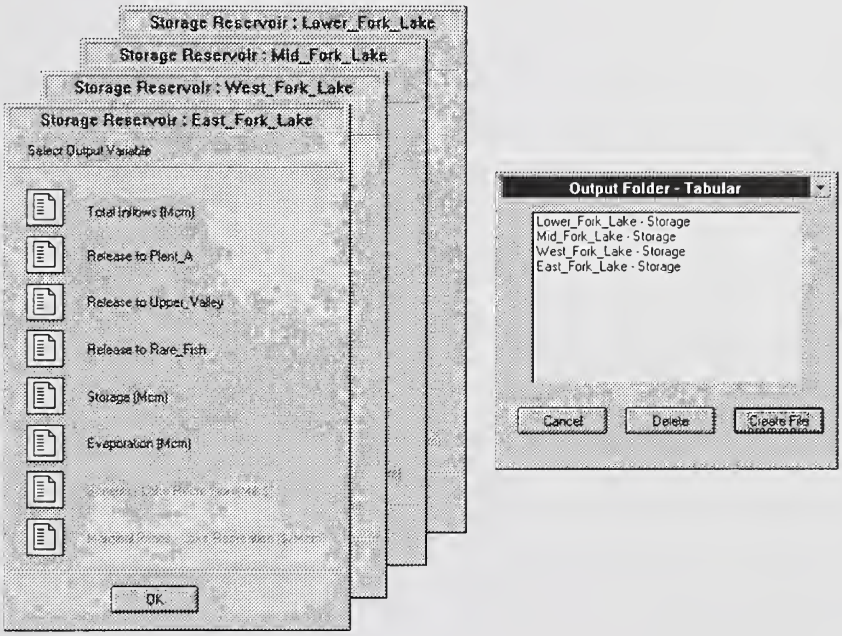


Figure 7.10 Specifying a list of variables for the tabular output folder.

In contrast to single-variable graph windows where information is updated during the sequential optimization process, graphs and tables created by the output folder reflect the state of the system at the moment the output is created (i.e., a static view). If only the optimal state of the network is of interest, the output folder should be created after the optimization. Printouts of any of the windows, including the NWS, can be obtained by making the window of interest the active window and selecting Print from the File Menu. The Print command also provides most of the customary Windows print settings.

Chapter 8

Model Applications

This chapter presents two hypothetical river basin systems of average complexity to demonstrate basic model applications at a monthly time step. Hypothetical systems were chosen over real systems for demonstration purposes.

Description of the River Basin System

The first hypothetical river basin is represented by the flow network in figure 8.1, in which the water flows from left to right. The basin comprises three water source subbasins, the East Fork, West Fork, and Lower Fork. Two of the subbasins are headwater catchments, at high elevations (left portion of the graph). The third catchment area (Lower Fork) delivers unregulated inflow to the downstream portion of the main water course. Flows are assumed known for the entire study period. The long-term mean-annual discharges for the three catchments were estimated as 9,600 Mcm (East Fork), 6,700 Mcm (West Fork), and 6,500 Mcm (Lower Fork). The flow regime is characterized by highly peaked hydrographs, typical of mountainous regions where runoff is dominated by snowmelt. Flows are relatively low during the winter and very high during the spring and early summer months (i.e., April through July).

The natural flows in the system are regulated by four reservoirs: East Lake, West Lake, Mid Lake, and Lower Lake (figure 8.1). Table 8.1 lists the operational characteristics of the reservoirs. The upper reservoirs (East and West Lakes) have relatively limited capacity to regulate inflows, with active storage/inflow ratios of about 0.3. The third reservoir, Mid Lake, which impounds water from the two upstream river forks, has a storage capacity equal to 70% of the mean-annual discharge at that site. The most downstream reservoir also has limited (medium to low) capacity (storage to inflow ratio equal to 0.26), but around 2/3 of its inflows are pre-regulated. When considering the whole basin, the ratio of active storage to natural flows is approximately 1.0. The system contains enough active storage to accommodate the total amount of water generated by the three catchments during an average hydrologic year.

Table 8.1 Operational characteristics of four hypothetical reservoirs.

Reservoir	Storage [Mcm]			Sto./Inf.	Elev. vs. Stor		Area vs. Stor		Constraints		
	Min.	Max.	Init.= Fin.	Ratio	c_1	d_1	c_2	d_2	Min.	Max.	Final
East	2,000.	5,000.	3,500.	0.31	1.86	0.44	1.28	0.59	y	y	y
West	2,000.	4,000.	3,000.	0.30	2.77	0.48	0.26	0.75	y	y	y
Mid	6,000.	17,500.	14,000.	0.70	3.03	0.38	0.30	0.73	y	y	y
Lower	14,000.	20,000.	17,500.	0.26	0.79	0.44	1.00	0.68	y	y	y

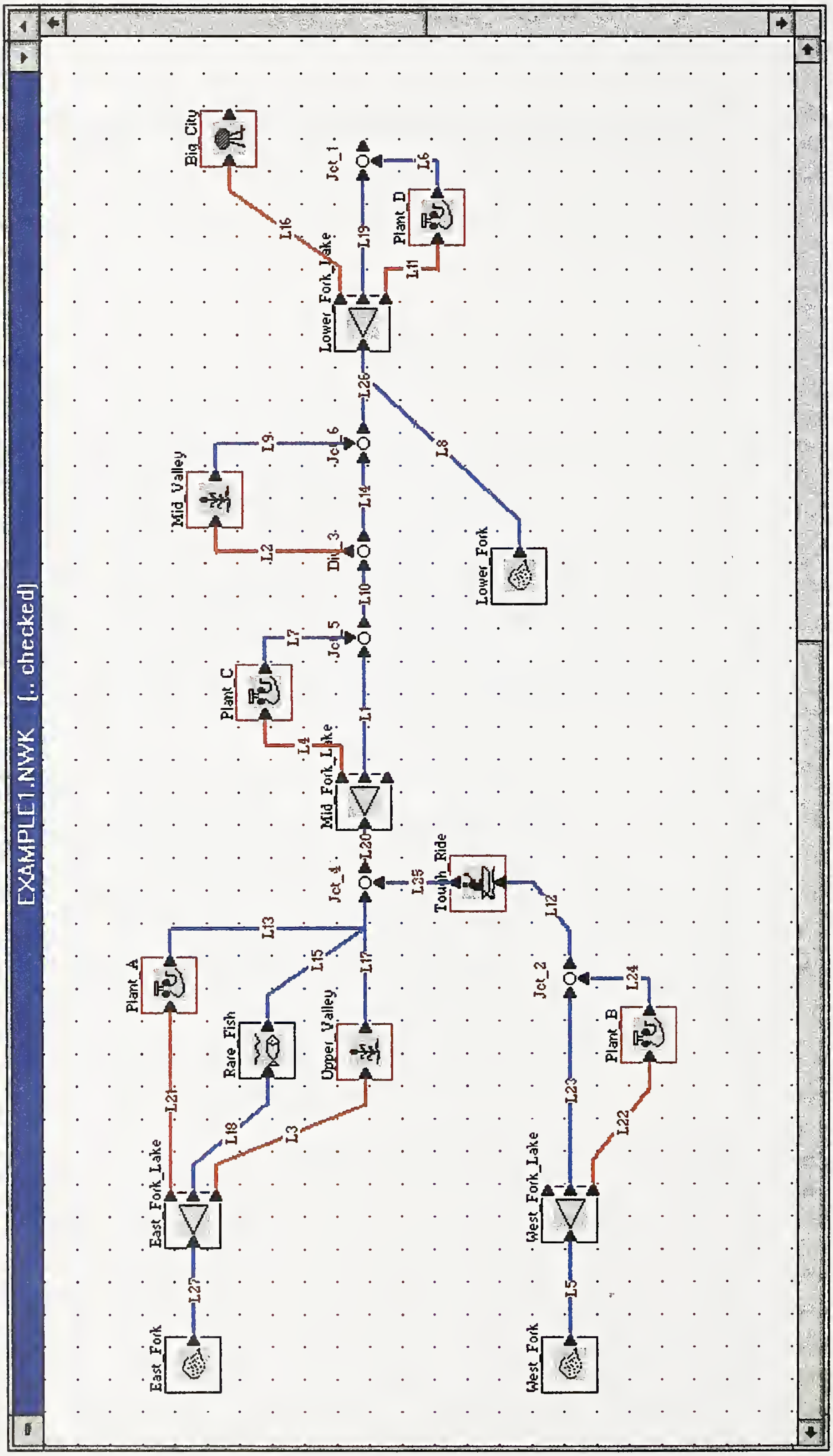


Figure 8.1 Flow network of an hypothetical river basin system.

Table 8.1 also provides information about the elevation-storage and area-storage relations for the four reservoirs (equations (3.4a) and (3.4b)). Values of net evaporation (monthly rates) are assumed known based on data from meteorological stations located in the study area. Monthly rates were assumed identical for all reservoirs and assumed equal to 30, 35, 40, 50, 65, 70, 85, 70, 60, 50, 40, and 35 mm/month, from January through December, respectively. Data pertinent to the spillways are set to zero (not used for operation at monthly time intervals). The three operational constraints for reservoirs, minimum, maximum, and final storage, are enforced according to table 8.1.

Hydropower is generated using releases from the four reservoirs. Due to existing topographical conditions, the two high elevation reservoirs, East Lake and West Lake, are connected to run-of-river powerplants, A and B, by canals that run parallel to the river at an almost constant elevation. It is reasonable to assume that the two power facilities operate under fixed-head conditions (figure 3.4, right side). The remaining two powerplants, C and D, are located in the intermediate and lower portions of the system, operate directly connected to the reservoir, and are subject to variable hydraulic heads (figure 3.4, left side). Table 8.2 describes the power facilities.

Table 8.2 Characteristics of four hypothetical hydropower facilities.

Utility	Installed Capacity [Mw]	Design Discharge [m3/s]	Group Efficiency [-]	Scheduled Maint.	-- Energy Rate -- a_1 [Mwh/Mcm]	b_1	Plant Release Min. [Mcm]	Max.	Constraints Min.	Max.
Plant A	350	920.	0.90	no	97.15	0.00	100	≈2,400.	yes	yes
Plant B	300	600.	0.90	no	164.13	0.00	100	≈1,600.	yes	yes
Plant C	450	800.	0.90	no	163.76	0.04	100	≈2,100.	yes	yes
Plant D	250	1,600.	0.90	no	89.66	0.02	100	≈4,200.	yes	yes

We assume uninterrupted service for maintenance or repairs of the generation groups (all monthly coefficients are equal to one). Moreover, minimum powerplant releases (discharge per month) are entered as indicated in table 8.2. Maximum releases are automatically computed by the model from the specified powerplant design discharge and maintenance schedule coefficient (by clicking the Reset button in the Maximum Releases dialog box). Maximum and minimum powerplant releases are enforced in the formulation of the problem, by enabling the corresponding constraints in the Operational Constraint box.

Two separate outlet works in East Lake Dam control releases from the reservoir, one on each bank. The left-bank reservoir outlet conveys water into the run-of-river hydroelectric Plant A. Characteristics of the powerplant are in table 8.2. The right-bank outlet discharges into a canal that conveys water to the Upper Valley agricultural area. Estimations are that 50% of the irrigation water returns to the natural channel via ground water accretion, far downstream from the dam. Minimum and maximum water allocations for the Upper Valley zone are in table 8.3. The operational constraints for the offstream demand areas (minimum, maximum, and seasonal) are disabled for this exercise.

Table 8.3 Offstream and instream demand areas for the hypothetical system.

Demand Area	Demand Min. [Mcm]	Demand Max. [Mcm]	Return Flow Coefficient [-]	Constraints Min.	Constraints Max.	Constraints Seas.
Upper Valley	0	1000.	0.5	no	no	no
Mid Valley	0	2000.	0.3	no	no	no
Big City	0	1000.	0.0	no	no	no
Rare Fish	var.	4000.		yes	no	no
Tough Ride	var.	4000.		no	no	no

The central link extending downstream from East Lake reservoir (figure 8.1, link 18), represents the natural riverbed. This portion of the river was known for the presence of native fishes whose populations have declined since reservoir construction, thus, becoming an environmentally sensitive area. Flow requirements were determined using a flow-habitat model as part of a fish recovery program. The recommended mean-monthly flows for the fish habitat protection area (figure 8.1, Rare Fish) are 20, 20, 20, 30, 40, 60, 70, 50, 30, 20, 20, and 20 Mcm for the months of January through December, respectively. The recommended minimum releases are aimed at reestablishing some of the natural variability of the flow regime.

The West Lake reservoir regulates natural flows contributed by the West Fork River subbasin. The reservoir supplies water for the run-of-river Powerplant B, located on the right-bank of the river. Characteristics of the plant are in table 8.2. The river reach downstream from West Lake but upstream of the junction with the East Fork is used extensively for recreational purposes, mostly rafting and kayaking. This water use is represented in the flow network by the instream Tough Ride user (figure 8.1). Operation of the commercial recreational activities is from May through September.

Regulated flows from the East and West Fork Rivers enter Mid Lake reservoir, where they are regulated again. Releases from Mid Lake Dam are used to generate power at Plant C, which is a variable-head powerplant. Characteristics of Plant C are in table 8.2. Turbine flows are returned to the river at the toe of the dam. Further downstream, in the middle portion of the basin, water is diverted to an irrigation demand zone named the Mid Valley agricultural area. Flow measurements indicate that around 30% of the diverted water returns to the stream (table 8.3), which implies a 70% consumptive use. Maximum and minimum irrigation diversions are specified in table 8.3, although, as for all offstream water uses, no operational constraints are enforced in this exercise.

Controlled flows from Mid Lake (minus Mid Valley consumptive use) and the Lower Fork tributary enter Lower Lake reservoir. There are two controlled releases from Lower Lake Dam into Plant D, which is a variable-head powerplant (table 8.2), and into a canal that conveys water to a municipal and industrial demand area named Big City. Big City is a transbasin diversion from which no return flows are expected, as evidenced by the lack of a return link from Big City.

Figure 8.1 shows the names of the system components as entered by the user, and the numbers assigned automatically by the model to links and nodes (shown only for junctions and diversions). The model may randomly change the numbering of links and nodes every time the connectivity of the network is updated. A change in the numbering of nodes and links implies a possible change in the mathematical arrangement of the system components when the optimization problem is formulated (Chapter 6, Mathematical Connectivity of System Components). Slightly different optimal solutions should be expected from different mathematical network arrangements. Although more research is needed, it is believed that numerical precision in the computations is responsible for this behavior. Also, as mentioned in Chapter 7, Selecting the Optimization Technique, changes in the prescribed maximum number of iterations per quadratic programming sequence may also cause slight changes in the final optimal solution.

Once network connectivity is updated (indicated by *...checked* at the top of the figure), seven links turn from blue to red. Red links indicate the location in the network where decision variables are created by the model to formulate the water allocation problem. Similarly, red lines framing some water users indicate that they economically compete for water.

Defining Demand Functions

The demand functions used in this example reflect the general structure of prices in Chapter 4, Benefit Functions. Average annual and monthly prices for energy are assumed equal for all power facilities (i.e., no change in energy price from month to month). This is a reasonable assumption considering that all power powerplants sell their energy to the same electrical grid system. Furthermore, downward slopping demand curves, equation (4.3), reflect the difference in energy prices from on-peak to off-peak hours where the rate of decay is constant for all months. Table 8.4 has information on hydropower economics.

Besides hydropower, the only other instream user economically competing for water is the water-based recreation area Tough Ride. This highly seasonal activity, running from May through September, is represented by linear demand functions, equation (4.25). The seasonality can be inferred from the values of the coefficients of the price functions in table 8.5, which shows values different from zero only for May through September.

Demands are also seasonal for the two irrigation areas, Upper Valley and Mid Valley, with growing seasons from April through October. The coefficients of the exponential decreasing price functions, equation (4.15), are shown in table 8.5. Prices are equal to zero for the nonirrigation months. For the transbasin urban demand zone, Big City, prices also decay exponentially with quantity supplied, equation (4.20), although, in contrast to the agricultural zone, municipalities and industries consume water all year around. This is indicated in table 8.5 by the nonzero values of demand function coefficients for all months of the year.

Table 8.4 Information on hydropower economics.

Hydroplant	Avg. Energy Price P [$/MWh]	P_s	Price Function a_2 [$/MWh]	b_2
Plant A	30.	30.	46.75	1,712.
Plant B	30.	30.	46.75	2,085.
Plant C	30.	30.	46.75	2,917.
Plant D	30.	30.	46.75	800.

Table 8.5 Economic information on instream and offstream demand areas.

Month	Tough Ride a_5 [$/Mcm]	b_5	Upper Valley a_3 [$/ Mcm]	b_3	Mid Valley a_3 [$/Mcm]	b_3	Big City a_4 [$/Mcm]	b_4
Jan	0.	0.	0.	0.	0.	0.	38,700.	78.
Feb	0.	0.	0.	0.	0.	0.	49,800.	92.
Mar	0.	0.	0.	0.	0.	0.	65,200.	118.
Apr	0.	0.	40,000.	662.	11,700.	207.	71,200.	134.
May	2000.	2.0	40,000.	749.	15,200.	320.	76,000.	145.
Jun	2000.	2.0	40,000.	814.	18,200.	395.	79,600.	155.
Jul	2000.	2.0	40,000.	857.	18,200.	600.	85,000.	164.
Aug	2000.	2.0	40,000.	890.	18,200	634.	83,100.	160.
Sep	2000.	2.0	40,000.	749.	16,000.	485.	73,400.	141.
Oct	0.	0.	40,000.	543.	12,700.	358.	67,800.	115.
Nov	0.	0.	0.	0.	0.	0.	56,700	88.
Dec	0.	0.	0.	0.	0.	0.	41,500.	71.

Finding an Efficient Water Allocation

This section describes the water allocation for the hypothetical river basin that was found given the water prices and operational constraints described earlier. The problem was solved over a two-year period using monthly time intervals. The optimization horizon was also set equal to 24 months, which implies that the quasi-continuous optimization procedure was unnecessary.

Because only a minimum number of operational constraints was specified for the network, the model finds a water allocation based almost exclusively on the water value. After this baseline case is solved, conditions are modified for different portions of the network to illustrate specific model capabilities.

Initial Feasible Solution

The flow cascading approach (Chapter 5) provides the initial feasible solution (IFS) for the specific problem. The approach consists of simply cascading flows through the reservoirs, avoiding storage regulation when possible, and assigning water to instream and offstream users provided that reservoirs' and water users' operational constraints are not violated. Figure 8.2 displays some selected graphical outputs, created by the model, which illustrate the state of the river-reservoir system after the IFS was found.

The storage trajectories for the reservoirs indicate constant reservoir levels for two of the four reservoirs. Figure 8.2 shows monthly releases from East Lake Dam Plant A, Upper Valley, and Rare Fish. The IFS algorithm satisfies minimum required flows at the Rare Fish river reach and then arbitrarily chooses any competing water user, Plant A in this example, to allocate the remaining flows. For months in which the powerplant capacity was exceeded (months 5, 6, and 16), the extra reservoir inflows were directed toward the Upper Valley irrigation area. Release conditions from Mid Lake are different from those at East Lake in that a single user (Plant C) is directly linked to the reservoir. Because the maximum capacity of the powerplant is at times exceeded, the reservoir is forced to regulate its inflows to avoid spillages.

When encountering offstream demand areas that claim water directly from the natural channel, rather than from a storage reservoir as Upper Valley does, and when no minimum flows are enforced, the IFS algorithm maintains water in the river rather than diverting it. Figure 8.2 shows no water allocated to Mid Valley agricultural zone.

The value of the total benefits from this arbitrary pattern of water allocation over two years of operation was calculated by the model as $1,054.6 million. Although this initial solution is feasible, it is not an efficient allocation of water, as demonstrated next.

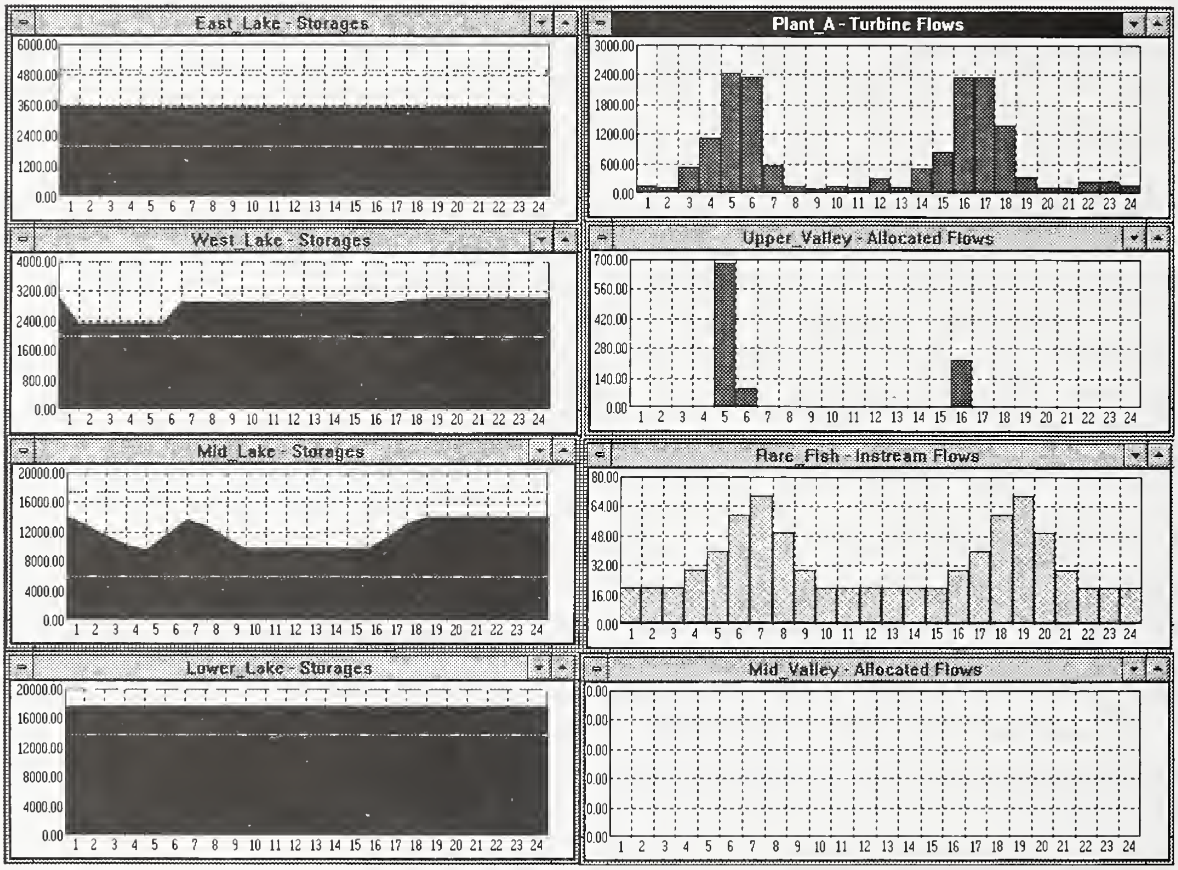


Figure 8.2 Selected graphical outputs from the initial feasible solution.

Final Solution

A crucial element during the allocation process is the monotonic decrease in the marginal price of allocated water with the increasing availability of flows. Reservoirs are also important during the allocation of water in the basin because flow regulation allows for releases that efficiently satisfy instream and offstream demands. Figure 8.3 shows the high contrast between the uncontrolled inflows and the controlled outflows from the West Lake reservoir.

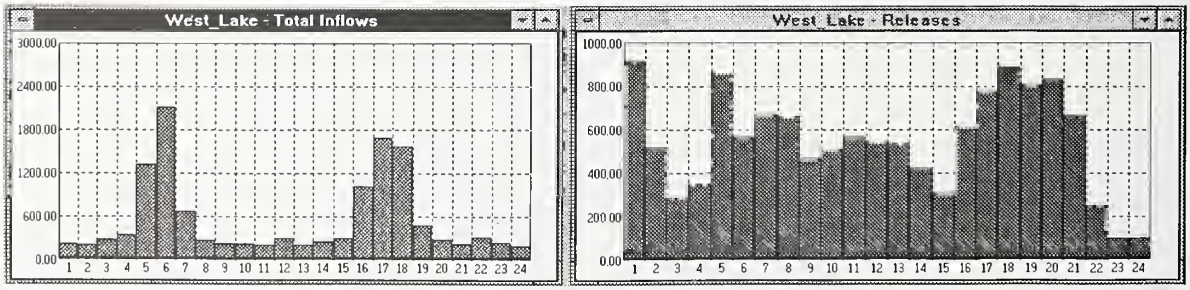


Figure 8.3 Uncontrolled inflows and controlled outflows from West Lake.

Figure 8.4 shows selected graphs of the result of the optimization. The two headwater reservoirs absorb most of the variability of the natural inflows creating favorable conditions for the rest of the basin to generate hydropower and distribute water during the seasons in which it is needed. Water levels in the East and West Lake reservoirs oscillate from empty to full to avoid spilling the large volumes of inflow associated with snowmelt. Because Plants A and B are run-of-river plants (i.e., their energy generation is not a function of reservoir storage levels), they impose no limitations in the oscillation of the water levels in the two headwater reservoirs. Contrarily, at Mid Lake Dam, a variable-head powerplant, almost constant and maximum water levels are maintained during the whole optimization horizon to maximize hydropower production. High reservoir levels are also maintained at Lower Lake, which besides receiving the regulated inflows from the upstream portion of the basin, also captures uncontrolled inflows from the Lower Fork Basin forcing an appreciable degree of regulation at the reservoir.

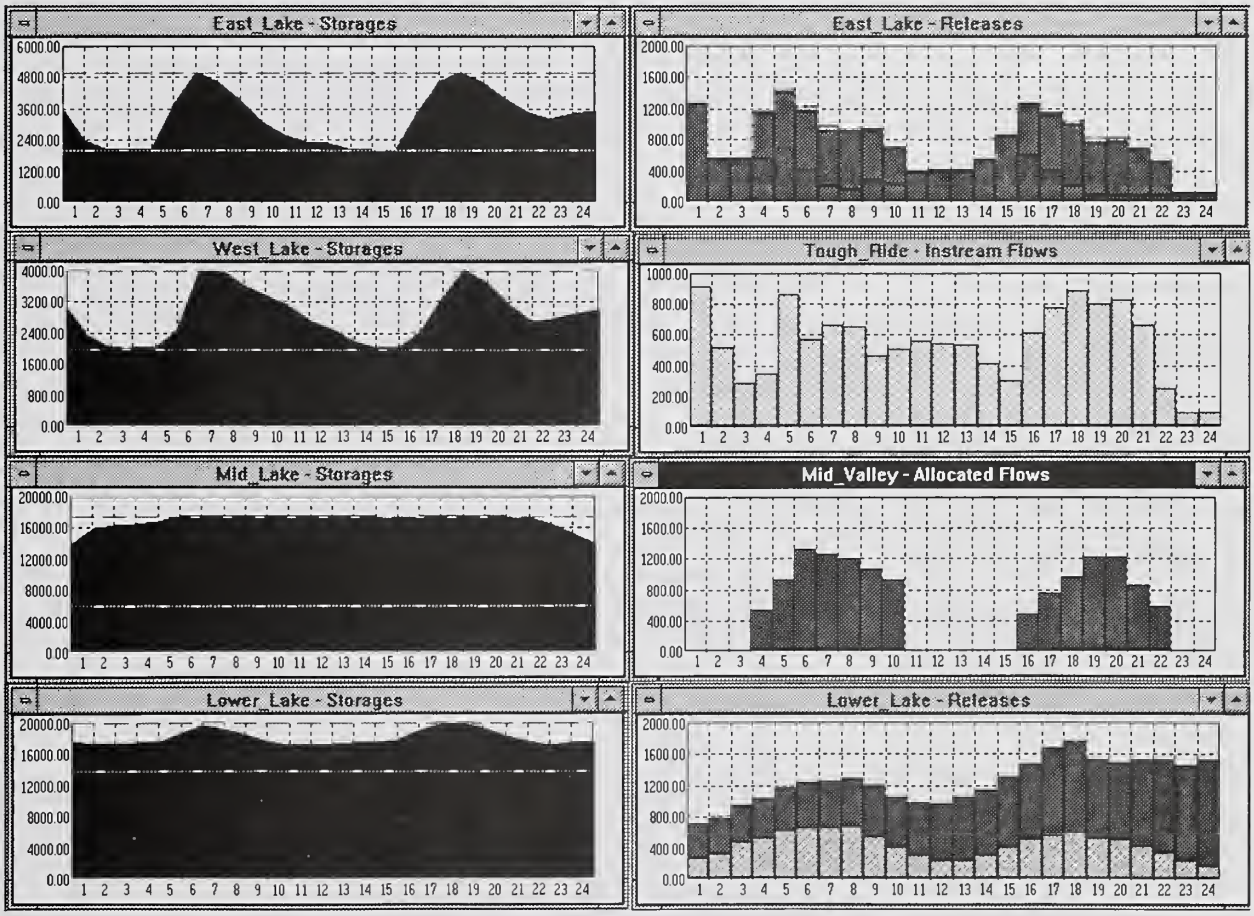


Figure 8.4 Selected graphical outputs from the final optimization solution.

Water diversions into the Mid Valley irrigation zone are in figure 8.4. The differing demand functions for each month of the growing season are responsible for the seasonal distribution of the allocated water since no constraints were enforced for any instream or offstream water use during this exercise. The zero diversions during months outside the irrigation season are the result of specifying zero economic benefits for those months (table 8.5).

Optimal releases from Lower Lake are displayed by the stacked bar diagram in the lower right-hand corner of figure 8.4. The lower bars correspond to water supplied to the Big City M&I demand area; the upper bars represent water released for hydropower to Plant D. The seasonal variability in marginal prices assigned for Big City (see table 8.5) translates into a pattern of water allocation with similar seasonality. Power releases from Plant D are small as the reservoir builds up storage and then increase during the second year as more water is available.

Releases from West Lake, after passing through Plant B, become instream flows in the West Fork River where the Tough Ride recreation area is located. Figure 8.4 indicates the effort made by the model to release relatively larger flows from West Dam during periods 5 through 8 and 17 through 20, the months of rafting and kayaking use. The upper bar graph in figure 8.4 shows the releases from East Lake. Optimal flows for Upper Valley display the classical seasonality of irrigation demand, whereas Plant A releases are more uniformly distributed over the optimization horizon because they tend to follow the storage fluctuations in the reservoir. Link 18, the river reach where the fish protection area was established, received no more flows than the ones enforced by the user via the minimum flow constraint. This is expected since the optimization algorithm gives preference for releases to the users generating revenues. The Rare Fish system component has no economic benefits attached to it.

Evaporation losses represent a small amount of water when compared to the total inflows to the reservoirs. The maximum losses occur at Lower lake, approximately 2%. Evaporation losses do not influence the storage patterns. Reservoir spillages do not occur for the period of record analyzed in this exercise.

As illustrated in figure 5.1, the optimization algorithm SQP searches for the optimal allocation of water by successively solving quadratic programming problems. Figure 8.5 shows the evolution of the value of the global objective function as it progresses toward the optimal solution for the specific problem. There is a substantial improvement in the water

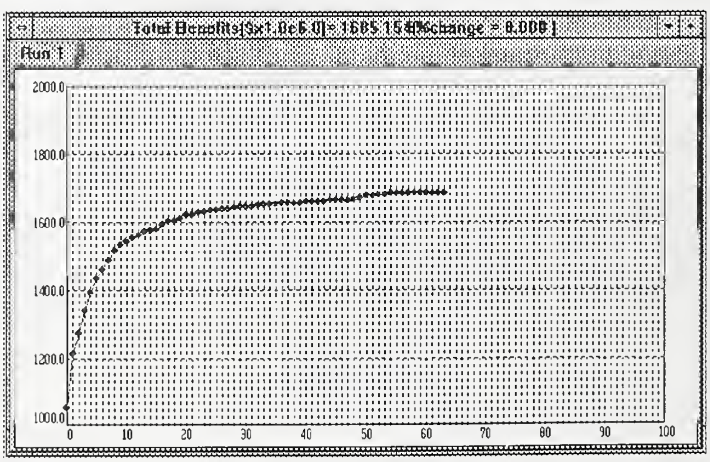


Figure 8.5 Evolution of the value of the global objective function.

allocation during the first few sequences, accompanied by a drastic increase in revenues, followed by a relatively flat portion of the revenue curve where only small changes occur during the remaining sequences. The search for the optimum is stopped either when the maximum number of sequences (100 in this example) or the accuracy parameter (figure 7.7) is reached; both parameters are specified by the user. At the optimum, the total revenues computed by the model amounted to $1,685.2 million over the two-year period. The model was allowed to continue the sequential optimization until the difference in global return between two consecutive QP sequences was less than $10 thousand; requiring 62 sequences, with 14 iterations/sequence to reach that point. This permitted proper comparisons with additional runs of the model presented in the following subsections.

Changes in Offstream Water Demand

The allocation of water to the offstream demand areas (irrigation and M&I) for the baseline case relied exclusively on the demand curves specified for each time period. For instance, the optimal allocation for Mid Valley (figure 8.4, third bar graph) displays a seasonal variability consistent with the monthly demand functions in table 8.5.

In this exercise we introduce a change in the demand of water from an offstream zone, demonstrate how the new problem can be solved with minimum computational effort, and discuss the implications of the change. Flows are required for the nonirrigation month January at Mid Valley. The new use will be incorporated into the model by altering the economic conditions in the network rather than by imposing a minimum flow level (constraint). For the baseline case the demand function had null parameters for January (table 8.5); now they are similar to those for April (a_3=10,000 and b_3=250).

The user can solve the new water allocation problem by altering the marginal price of water at Mid Valley for January, validating the network connectivity, finding a new IFS, and solving the optimization problem to find the efficient water allocation policies under the new economical condition. However, because changes in the price structure does not invalidate feasibility in the network, the user can proceed from the last optimal solution, in this case the baseline solution, to the new optimal condition.

Changes in marginal prices can be introduced any time during the execution of the optimization or at the end of the sequences. If the change is introduced as the application is executing, the graphical outputs will indicate how the model immediately starts redirecting the allocation of water to accommodate the new economic conditions. If the change is introduced after AQUARIUS has reached an optimum, the model will adopt the last optimal solution as the IFS for the new round of optimization. For a minor change in price, the model will find the new optimum very quickly.

Figure 8.6 shows the results of the above outlined procedure. The water allocation at Mid Valley shows positive flows for January (compare with the baseline solution in figure 8.4). The allocation of flows to Mid Valley during January reflects the willingness of the irrigation area to compensate other users for the extra water demanded for that month. In other words, a new Pareto optimal arrangement was found.

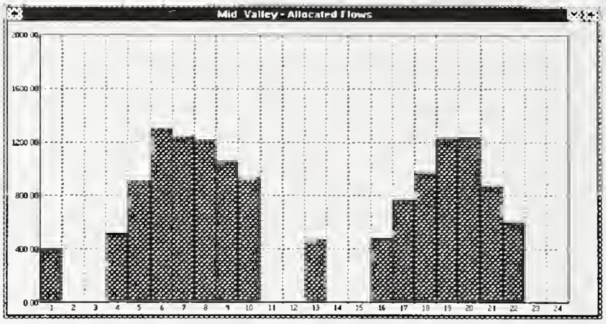


Figure 8.6 Change in offstream water demand.

An additional example concerning offstream demand areas is reported here, although the change relates to physical rather than economic data. The purpose of the exercise is to quantify the economic impact in the basin of an improvement in irrigation efficiency. Either by structural measures or conservation practices, the water duty for irrigation often can be decreased. While this may imply several modifications to the original baseline case, we will simply increase the amount of return flow that originates from the Upper Valley irrigation area from 0.5 to 0.7. This has practically the same effect on the network of reducing the water duty, since more water will be left in the river for use by other system components downstream.

After solving the network with the change indicated above (note that when a physical variable is changed a new initial feasible solution is required), the benefit from the network operation increases from \$1,685.2 million to \$1,712.5 million, a \$27.3 million (1.6%) change over the two-year period. In this example, the optimal point is attained using 13 iterations per sequence. Additional information can be extracted to identify the downstream water uses who benefited from this improved economic condition. The increase in benefit provides economic incentive for negotiations among the multiple users in the basin to finance the structural improvements necessary to conserve water in the basin.

Change in Energy Price

While unit revenues for hydroenergy are variable according to the type of energy being generated, on-peak vs. off-peak (figure 4.2), some power systems are compensated economically for the energy generated using a single rate, regardless of the time of the day in which the energy is delivered. The purpose of this exercise is to demonstrate how this case can be contemplated by AQUARIUS and to show the operational consequences of such an assumption.

Within the AQUARIUS formulation, adopting a constant unit price for energy is equivalent to replacing the exponential curve in equation (4.3) by a single step function (i.e., a constant value for all plant utilization factors). A constant price for energy can be simulated by simply adopting a very large number for the parameter b_2 in (4.3) (800,000 in this example) such that the

exponential term $exp(-T/b_2)$ becomes practically equal to one. Then, the value assigned to the parameter a_2 will represent the constant rate paid for hydroenergy. A constant price was adopted for one of the powerplants (Plant D, 25 $/MWh), while maintaining the rest of the hypothetical system. The system operation was reoptimized under these new conditions, and the values of the old and new powerplant releases from Plant D are in figure 8.7.

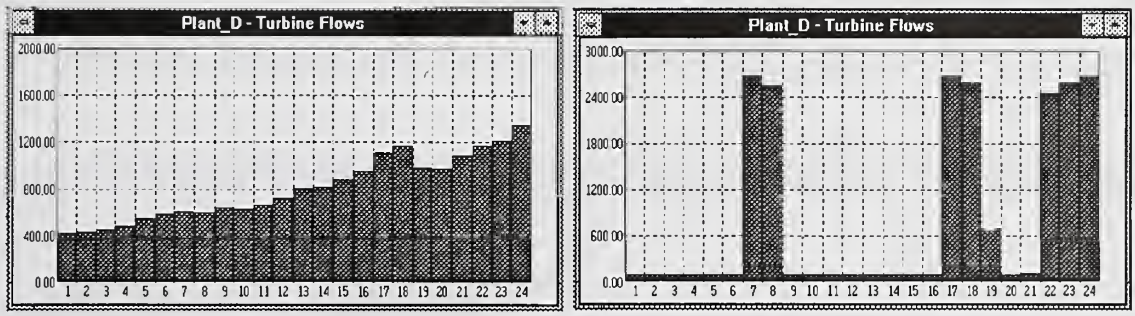


Figure 8.7 Hydropower releases with decreasing and constant energy price.

The model for the exponential price (left graph) indicates power releases to cover as much as possible of the on-peak volumes for all time steps, since on-peak releases have a higher price. As more water becomes available, releases during off-peak periods are also covered. When interpreting the series of releases from Plant D, remember that the upper portion of the basin is holding most of its releases during the first months to build up storage in the reservoirs.

In contrast to the above, when the energy price function for Plant D is a constant, the powerplant is not compelled to satisfy on-peak releases as before (figure 8.7, right graph). Although power releases occur in a more random pattern, they are more economically convenient for the system when considering the other reservoir functions (i.e., to gain storage whenever possible to maximize the amount of energy generated with the same amount of water and to satisfy the demand from Big City). This series of power releases would be unacceptable in a real system where, typically, minimum power releases would be imposed to ensure that a minimum amount of energy is generated each month. This is also a viable option using AQUARIUS.

Economic Efficiency for a Simple Case

In this exercise, we use a simple flow network to illustrate the basic concept of economic efficiency used in the model to drive the allocation of water in a river basin. This network consists of a single reservoir with two offstream demand areas, The Farm and The City, which withdraw water directly from the reservoir (figure 8.8, upper-left).

The irrigation and urban zones have totally different demands regarding the total amount of water required and their seasonal distribution. The Farm receives water only during the growing season, April through October, whereas The City receives water all year. Monthly price curves were

provided for the two water uses according to their seasonal demands. Furthermore, no minimum or maximum limits on releases were enforced. The only constraints were upper and lower bounds on the reservoir storage.

Figure 8.8 shows the most significant outputs from the optimal solution: the reservoir storage trajectory, the optimal releases, and the water marginal prices for the two uses computed at the point of optimality. As stated in Chapter 2, the model will allocate releases at a constant marginal price, provided that an unbounded solution to the water allocation problem is obtained.

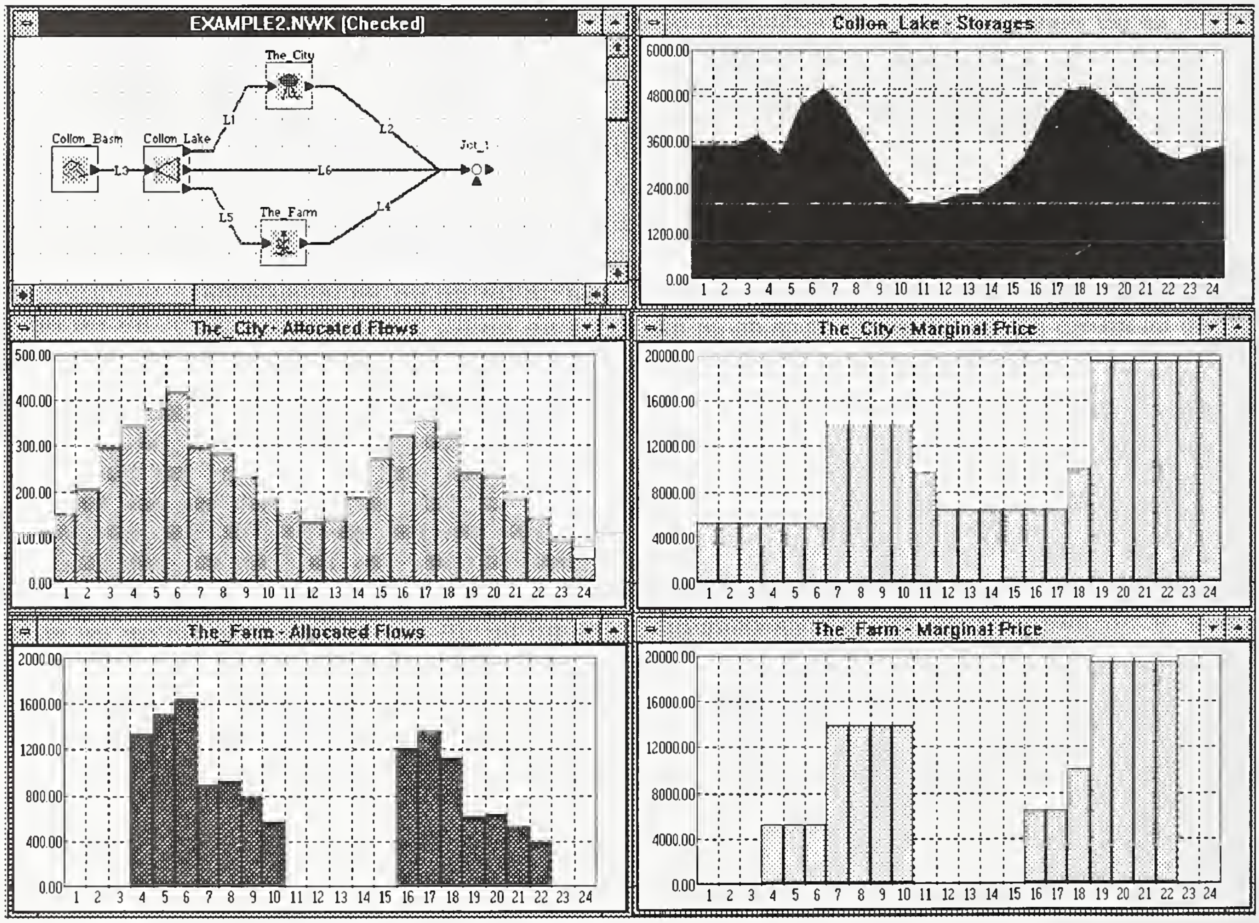


Figure 8.8 Conditions of economic efficiency in water allocation.

Figure 8.8 shows that the model reached the optimum when marginal prices were equalized for the two demands (compare the two marginal price bar graphs). Once the optimal solution has been reached, the value of water for each user, at the margin, is the same. This is not to say that the optimally allocated flows are equal for the two users. In fact, one can observe that the two users receive totally different amounts of releases.

Under optimal conditions, the model keeps the marginal prices constant in time provided that the solution of the problem is unbounded. Figure 8.8 shows that once the storage in the reservoir hits the upper or lower bound, the solution becomes bounded (i.e., it is not an interior solution anymore) and the marginal price changes. The solution of the reservoir problem is interior between time periods 1 through 6; thus, the marginal prices remain constant during those 6 months. Similar behavior can be observed for other subperiods with interior solutions (e.g., 7-10, 12-17, 19-24). This demonstrates, as discussed in Chapter 2, that the marginal benefit of the release must remain constant in time if the policy is to be optimal.

Software Performance

The time of execution of an application depends on the: length of the Period of Analysis, length of the Optimization Horizon, number of decision sets (red links in the network worksheet), number of operational constraints, and accuracy required for the optimal solution. The execution times in table 8.6 correspond to the solution of the Example1.nwk for different optimization horizons, running in a microcomputer with a 200Mhz Intel Pentium Pro processor. The execution times do not include graphical output display in between sequences, which may increase the execution time 10 to 15%, depending on the number of output windows.

Table 8.6 Optimization execution times.

Period of Analysis	Optimization Horizon	Execution Time [min:sec]
12	12	00:10
24	24	01:00
48	48	05:00

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Appendix A

Derivatives of the Benefit Functions

This appendix provides the analytical derivation of the first and second order partial derivatives of the benefit functions introduced in Chapter 4. The expressions in this section are part of a library of functions that are used for the assemblage of the gradient vector and Hessian matrix presented in Chapter 5, Method of Solution.

Variable-Head Hydropower Benefit Function

HPW^{VH} - First Partial Derivatives

The benefit function for a variable-head powerplant, equation (4.9), includes three types of decision variables: 1) the powerplant release (*T* variable), 2) controlled releases (*XR* term) other than the powerplant release, and 3) controlled inflows (*XI* term) (figures 3.1 and 4.1). These controlled outflows and inflows are part of equation (4.9a), and therefore subject to differentiation. Three cases of first order partial derivatives (FPD) follow.

Case T FPD with respect to powerplant release *T* for any time step *i*. The role of T_i in the FPD is reflected by the first two lines of (A.1). The release T_i also appears implicitly in (4.9a) as a reservoir outflow for time periods subsequent to *i*, from *i+1* to the end of the optimization horizon *np*. This portion of the derivative is given by the last term in (A.1).

$$\begin{aligned}\frac{\delta B^{HPW^{VH}}}{\delta T_i} = \eta \frac{p_i}{P}\frac{a_2 b_1}{2} \Big\{ & [2a_1/b_1 + 2S^o + 2\sum_{k=1}^{i-1}(UI_k + XI_k) - 2\sum_{k=1}^{i-1}(T_k + XR_k + MR_k + UR_k) \\ & + (UI_i + XI_i - XR_i - MR_i - UR_i)]\exp(-T_i/b_2) - [T_i \exp(-T_i/b_2)] \Big\} \\ & - \sum_{k=i+1}^{np} \eta \frac{p_k}{P} a_2 b_1 b_2 [1 - \exp(-T_k/b_2)]\end{aligned} \tag{A.1}$$

Case XR FPD with respect to any other controlled release $d_{A_i}, d_{B_i}, ..$ from the same reservoir that supplies water to the powerplant under consideration. For example, the FPD in (A.2) is written with respect to the decision variable d_A. The controlled releases in the FPD at any given time step *i* is reflected by the first term of (A.2). Moreover, the $d_{A_i}, d_{B_i}, ..$ variables also appear as reservoir outflows for time steps *i+1* and forward. This is reflected by the second term.

$$\frac{\delta B^{HPW^{VH}}}{\delta d_{A_i}} = -\eta \frac{p_i}{P} \frac{a_2 b_1 b_2}{2} [1 - \exp(-T_i / b_2)] - \sum_{k=i+1}^{np} \eta \frac{p_k}{P} a_2 b_1 b_2 [1 - \exp(-T_k / b_2)] \qquad (A.2)$$

Case XI FPD with respect to any of the upstream decision variables $d_A^u, d_B^u, ..$ flowing into the reservoir that regulates flows for the powerplant of interest. For example, the first part of (A.3) reflects the role of d_A^u in the FPD at a given time step *i*. Moreover, the variables $d_A^u, d_B^u, ..$ also appear as reservoir inflows in (4.9a) for time *i+1* and forward. This is reflected by the second term.

$$\frac{\delta B^{HPW^{VH}}}{\delta d^u_{A_i}} = \left\{ \eta \frac{p_i}{P} \frac{a_2 b_1 b_2}{2} [1 - \exp(-T_i / b_2)] + \sum_{k=i+1}^{np} \eta \frac{p_k}{P} a_2 b_1 b_2 [1 - \exp(-T_k / b_2)] \right\} c_A^u \qquad (A.3)$$

where c_A^u is the coefficient of the upstream decision variable d_A^u (controlled inflow) for which the FPD was written.

HPW^{VH} - Second Partial Derivatives

The second order partial derivatives (SPD) presented in this subsection are from equations (A.1), (A.2), and (A.3). In each case we provide a reference to the relative position of the SPD within the global Hessian matrix as discussed in Chapter 6, Mathematical Connectivity of System Components.

Case T-T FPD and SPD with respect to the same variable *T*. Three sub-cases are considered:

1. component submatrix—diagonal terms

$$\frac{\delta^2 B^{HPW^{VH}}}{\delta T_i \, \delta T_i} = -\eta \frac{p_i}{P} \frac{a_2 b_1}{b_2} (a_1/b_1 + b_2/2 + \bar{S}_i) \exp(-T_i / b_2) \qquad (A.4)$$

2. component submatrix—off-diagonal terms (lower-triangular)

$$\frac{\delta^2 B^{HPW^{VH}}}{\delta T_i \, \delta T_j} = -\eta \frac{p_i}{P} a_2 b_1 \exp(-T_i / b_2) \qquad .... for\ j < i \qquad (A.5)$$

3. component submatrix—off-diagonal terms (upper-triangular)

$$\frac{\delta^2 B^{HPW^{VH}}}{\delta T_i \, \delta T_j} = -\eta \frac{p_j}{P} a_2 b_1 \exp(-T_j / b_2) \qquad .... for\ j > i \qquad (A.6)$$

Case T-XR FPD with respect to variable T, and SPD with respect to other controlled reservoir release, represented here by d_A. Two sub-cases are considered:

1. link submatrix—diagonal terms

$$\frac{\delta^2 B^{HPW^{VH}}}{\delta T_i \, \delta d_{A_i}} = -\eta \frac{p_i}{P} \frac{a_2 b_l}{2} \exp(-T_i / b_2) \tag{A.7}$$

2. link submatrix—off-diagonal terms (lower-triangular)

$$\frac{\delta^2 B^{HPW^{VH}}}{\delta T_i \, \delta d_{A_j}} = -\eta \frac{p_i}{P} a_2 b_l \exp(-T_i / b_2) \qquad \text{....... } for\ j < i \tag{A.8}$$

Case XR-T FPD with respect to other controlled reservoir release d_A, and SPD with respect to T. Two sub-cases are considered:

1. link submatrix—diagonal terms

$$\frac{\delta^2 B^{HPW^{VH}}}{\delta d_{A_i} \, \delta T_i} = -\eta \frac{p_i}{P} \frac{a_2 b_l}{2} \exp(-T_i / b_2) \tag{A.9}$$

2. link submatrix—off-diagonal terms (upper-triangular)

$$\frac{\delta^2 B^{HPW^{VH}}}{\delta d_{A_i} \, \delta T_j} = -\eta \frac{p_j}{P} a_2 b_l \exp(-T_j / b_2) \qquad \text{....... } for\ j > i \tag{A.10}$$

Case T-XI FPD with respect to T and SPD with respect to a controlled reservoir inflow, for example d_A^u. Two sub-cases are considered:

1. link submatrix—diagonal terms

$$\frac{\delta^2 B^{HPW^{VH}}}{\delta T_i \, \delta d^u_{A_i}} = \eta \frac{p_i}{P} \frac{a_2 b_l}{2} \exp(-T_i / b_2) \ c_A^u \tag{A.11}$$

2. link submatrix—off-diagonal terms (lower-triangular)

$$\frac{\delta^2 B^{HPW^{VH}}}{\delta T_i \, \delta d^u_{A_j}} = \eta \frac{p_i}{P} a_2 b_l \exp(-T_i / b_2) \ c_A^u \qquad \text{....... } for\ j < i \tag{A.12}$$

where c_A^u is the coefficient of the upstream decision d_A^u.

Case XI-T FPD with respect to a controlled reservoir inflow, for example d_A^u, and SPD with respect to variable T. Two sub-cases are considered:

1. link submatrix—diagonal terms

$$\frac{\delta^2 B^{HPW^{VH}}}{\delta d^u_{A_i}\, \delta T_i} = \eta \frac{p_i}{P} \frac{a_2\, b_1}{2} \exp(-T_i / b_2)\; c^u_A \tag{A.13}$$

2. link submatrix—off-diagonal terms (upper-triangular)

$$\frac{\delta^2 B^{HPW^{VH}}}{\delta d^u_{A_i}\, \delta T_j} = \eta \frac{p_j}{P} a_2\, b_1 \exp(-T_j / b_2)\; c^u_A \qquad \text{.......} \textit{ for } j > i \tag{A.14}$$

where c_A^u is the coefficient corresponding to the upstream decision variable d_A^u (controlled inflow) for which the SPD was written.

Fixed-Head Hydropower Benefit Function

HPW^{FH} - First Partial Derivatives

The benefit function for a fixed-head powerplant, equation (4.14), admits only one case of first partial derivative:

Case XI FPD with respect to any of the upstream decision variables contained in the term T_i. For example, writing the derivative for the decision set d_A^u .

$$\frac{\delta B^{HPW^{FH}}}{\delta d^u_{A_i}} = \eta \frac{p_i}{P} a_1\, a_2 \exp(-T_i / b_2)\; c^u_A \tag{A.15}$$

where c_A^u is the coefficient of the upstream decision variable d_A^u.

HPW^{FH} - Second Partial Derivatives

The second partial derivatives from equation (A.15) should consider all possible combinations of the controlled inflows. Two sub-cases are contemplated:

Case XI 1. Component submatrix—diagonal terms. FPD and SPD with respect to the same variable, for instance d_A^u .

$$\frac{\delta^2 B^{HPW^{FH}}}{\delta d^u_{A_i}\,\delta d^u_{A_i}} = -\eta \frac{p_i}{P}\frac{a_1 a_2}{b_2}\exp(-T_i/b_2)\,(c^u_A)^2 \tag{A.16}$$

2. Component submatrix—diagonal terms. FPD and SPD taken with respect to different variables, for instance, d^u_A and d^u_B .

$$\frac{\delta^2 B^{HPW^{FH}}}{\delta d^u_{A_i}\,\delta d^u_{B_i}} = -\eta \frac{p_i}{P}\frac{a_1 a_2}{b_2}\exp(-T_i/b_2)\,c^u_A\,c^u_B \tag{A.17}$$

Irrigation Benefit Function

IRR - First Partial Derivatives

The benefit function for an agricultural demand area, equation (4.19), admits a single case of first partial derivative:

Case XI FPD with respect to any of the upstream decision variables contained in the term A_i. For instance, writing the derivative for the variable d^u_A .

$$\frac{\delta B^{IRR}}{\delta d^u_{A_i}} = a_{3_i}\exp(-A_i/b_{3_i})\,c^u_A \tag{A.18}$$

where c^u_A is the coefficient corresponding to the upstream decision variable d^u_A that flows into the irrigation zone.

IRR - Second Partial Derivatives

The second partial derivatives are derived from equation (A.18). Two sub-cases are contemplated:

Case XI **1.** Component submatrix—diagonal terms. FPD and SPD taken with respect to the same variable. In this case, the decision set d^u_A is used.

$$\frac{\delta^2 B^{IRR}}{\delta d^u_{A_i}\,\delta d^u_{A_i}} = -\frac{a_{3_i}}{b_{3_i}}\exp(-A_i/b_{3_i})\,(c^u_A)^2 \tag{A.19}$$

2. Component submatrix—diagonal terms. FPD and SPD taken with respect to two different control variables. For example, sets d_A^u and d_B^u .

$$\frac{\delta^2 B^{IRR}}{\delta d^u_{A_i} \, \delta d^u_{B_i}} = -\frac{a_{3_i}}{b_{3_i}} \exp(-A_i / b_{3_i}) \; c_A^u \; c_B^u \tag{A.20}$$

Municipal and Industrial Benefit Function

M&I - First Partial Derivatives

The benefit function for a municipal and industrial demand area, equation (4.24), admits a single case of first partial derivative:

Case XI FPD with respect to any of the upstream decision variables contained in the term D_i. For example, taking the derivative with respect to d_A^u we obtain

$$\frac{\delta B^{M\&I}}{\delta d^u_{A_i}} = a_{4_i} \exp(D_i / b_{4_i}) \; c_A^u \tag{A.21}$$

where c_A^u is the coefficient of the decision variable of interest.

M&I - Second Partial Derivatives

Second partial derivatives are derived from (A.21). Two cases are contemplated:

Case XI 1. Component submatrix—diagonal terms. FPD and SPD taken with respect to the same variable, for instance, d_A^u .

$$\frac{\delta^2 B^{M\&I}}{\delta d^u_{A_i} \, \delta d^u_{A_i}} = -\frac{a_{4_i}}{b_{4_i}} \exp(-D_i / b_{4_i}) \; (c_A^u)^2 \tag{A.22}$$

2. Component submatrix—diagonal terms. FPD and SPD taken with respect to two different control variables, for example, d_A^u and d_B^u .

$$\frac{\delta^2 B^{M\&I}}{\delta d^u_{A_i} \, \delta d^u_{B_i}} = -\frac{a_{4_i}}{b_{4_i}} \exp(D_i / b_{4_i}) \; c_A^u \; c_B^u \tag{A.23}$$

where now c_A^u and c_B^u are the coefficients corresponding to the two decision variables of interest.

Instream Water Recreation Benefit Function

IRA - First Partial Derivatives

The benefit function for an instream water recreation activity, equation (4.29), admits a single case of first partial derivative:

Case XI FPD with respect to any of the upstream decision variables contained in the term R_i. For example, writing the derivative with respect to d_A^u yields

$$\frac{\delta B^{IRA}}{\delta d_{A_i}^u} = (a_{5_i} - b_{5_i} R_i)\, c_A^u \tag{A.24}$$

where c_A^u is the coefficient corresponding to the decision variable d_A^u .

IRA - Second Partial Derivatives

The second partial derivatives are derived from equation (A.24). Two cases are contemplated:

Case XI 1. Component submatrix—diagonal terms. FPD and SPD taken with respect to the same variable. In this example d_A is used.

$$\frac{\delta^2 B^{IRA}}{\delta d_{A_i}^u\, \delta d_{A_i}^u} = -b_{5_i} (c_A^u)^2 \tag{A.25}$$

2. Component submatrix—diagonal terms. FPD and SPD taken with respect to two different control variables, for example, variables d_A^u and d_B^u .

$$\frac{\delta^2 B^{IRA}}{\delta d_{A_i}^u\, \delta d_{B_i}^u} = -b_{5_i}\, c_A^u\, c_B^u \tag{A.26}$$

where the coefficients c_A^u and c_B^u correspond to the decision variables of interest.

Appendix B

Fitting Exponential Demand Functions

This appendix presents the equations used by AQUARIUS to fit exponential inverse demand functions based on economic information provided by the user. Further discussion regarding this topic is in Chapter 2, Economic Demand Functions, General Concepts.

Cases Considered

Three cases are considered for the analytical fitting of demand curves, described below with the assistance of figure B.1:

Case I: prices and quantities are known at any two points along the demand curve. In general, the two points may represent: 1) the price at which quantity demanded falls to zero, $\{Q_1,P_1\}$ and 2) the quantity at which price equals the existing price, $\{Q_2,P_2\}$, respectively.

Case II: price and quantity are known at a single point, $\{Q_2,P_2\}$, in addition to the elasticity at that same point yielding the dimensionless slope ϵ_2.

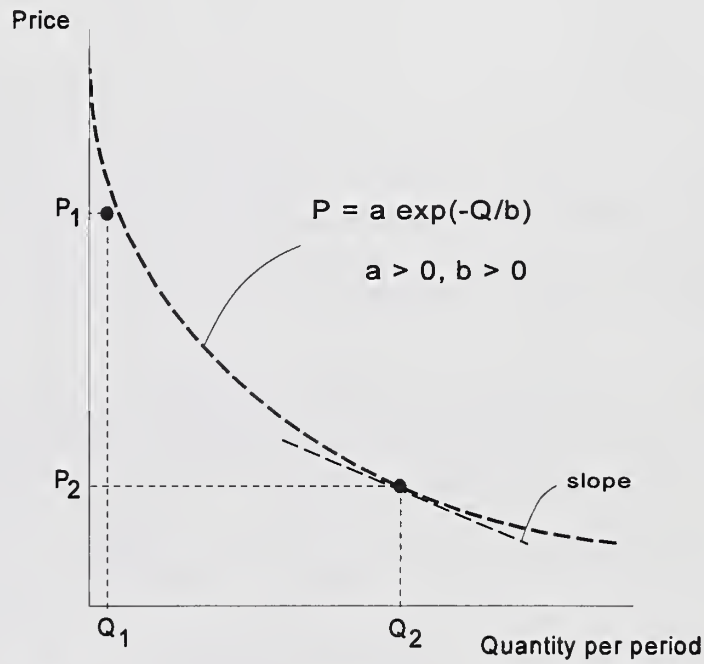

Figure B.1 Information defining an exponential demand.

Case III: is the combination of Cases I and II, in which three pieces of information are specified by the user: the two price-quantity points, $\{Q_1,P_1\}$ and $\{Q_2,P_2\}$, and the elasticity ϵ_2.

Basic Equations

We start the analysis by expressing the exponential inverse demand function $P=a\ exp(-Q/b)$ at the point $\{Q_1,P_1\}$, where demand falls near zero,

$$P_1 = a \exp(-Q_1/b) \tag{B.1}$$

Similarly, the exponential demand function can be written at the equilibrium point $\{Q_2,P_2\}$,

$$P_2 = a \exp(-Q_2/b) \tag{B.2}$$

By definition, the elasticity ϵ of the demand curve is given by Eq.(B.3) (Chapter 2, Economic Demand Functions, General Concepts). In particular, for an exponential demand function, elasticity is expressed by the ratio of the coefficient b and the quantity demanded Q,

$$\epsilon = \frac{\partial Q/Q}{\partial P/P} = -b/Q \tag{B.3}$$

Computing the Coefficients "*a*" and "*b*"

Each case is analyzed separately:

Case I Given the two paired values $\{Q_1,P_1\}$ and $\{Q_2,P_2\}$, the parameters a and b can be defined with no error since there is only one model of the form $P = a\ exp(-Q/b)$ that will assume the exact values $f(Q_1)$ and $f(Q_2)$. Working with equations (B.1) into (B.2) yields:

$$a = \frac{P_1}{\exp\left(\frac{\ln(P_1/P_2)}{1 - Q_2/Q_1}\right)} \quad \ldots\ldots \quad for \quad Q_1 > 0 \tag{B.4}$$

$$b = \frac{Q_2 - Q_1}{\ln(P_1/P_2)} \quad \ldots\ldots \quad for \quad Q_1 > 0 \tag{B.5}$$

If one of the points corresponds to the intercept $\{Q_1=0, P=P_1\}$, the coefficients a and b are given by:

$$a = P_1 \quad \ldots\ldots \quad for \quad Q_1 = 0 \tag{B.6}$$

$$b = \frac{Q_2}{\ln(P_1/P_2)} \qquad \text{..... } for\ \ Q_1 = 0 \tag{B.7}$$

Case II When one point of the curve is known $\{P_2, Q_2\}$ along with its elasticity ϵ_2, there is a unique solution to the problem. In this case, the coefficients a and b are expressed by:

$$b = -\epsilon_2 Q_2 \tag{B.8}$$

$$a = \frac{P_2}{\exp(1/\epsilon_2)} \tag{B.9}$$

Case III When two points and the elasticity ϵ at one of those points are known, the fitting problem becomes one of interpolating fitting since there are three pieces of information and only two parameters to estimate. The problem is reduced to one of finding the best estimates of the coefficients a and b of the exponential function $P = a\ exp(-Q/b)$ using the *Least-Squares* (LS) principle. This involves minimizing the sum of all squares of deviations of the observed points and elasticity from the fitted function.

When the economic information available, price-quantity points and elasticity, are thought to be of unequal reliability, the LS criterion can be modified to require that the squared error terms be multiplied by nonnegative weight factors w_i before the aggregated square error is calculated (*weighted least-squares*). That is, from (B.1), (B.2), and (B.3), the weighted sum of the square of the residuals assumes the form:

$$Min \quad S = w_1 [P_1 - a \exp(-Q_1/b)]^2 + w_2 [P_2 - a \exp(-Q_2/b)]^2 + w_\epsilon [\epsilon_2 - b/Q_2)]^2 \tag{B.10}$$

where the weighting coefficients w_1, w_2, and w_ϵ, for the two data points and elasticity, respectively, are a measure of the degree of precision or degree of importance of the three pieces of information in determining the coefficients of the demand function. Equation (B.10) is a nonlinear expression that can be minimized using different techniques.

Fitting Example

Figure B.2 shows the dialog-box presented to the user, which is accessed from the Tools menu, to estimate the coefficients of the exponential demand function for the three cases described above.

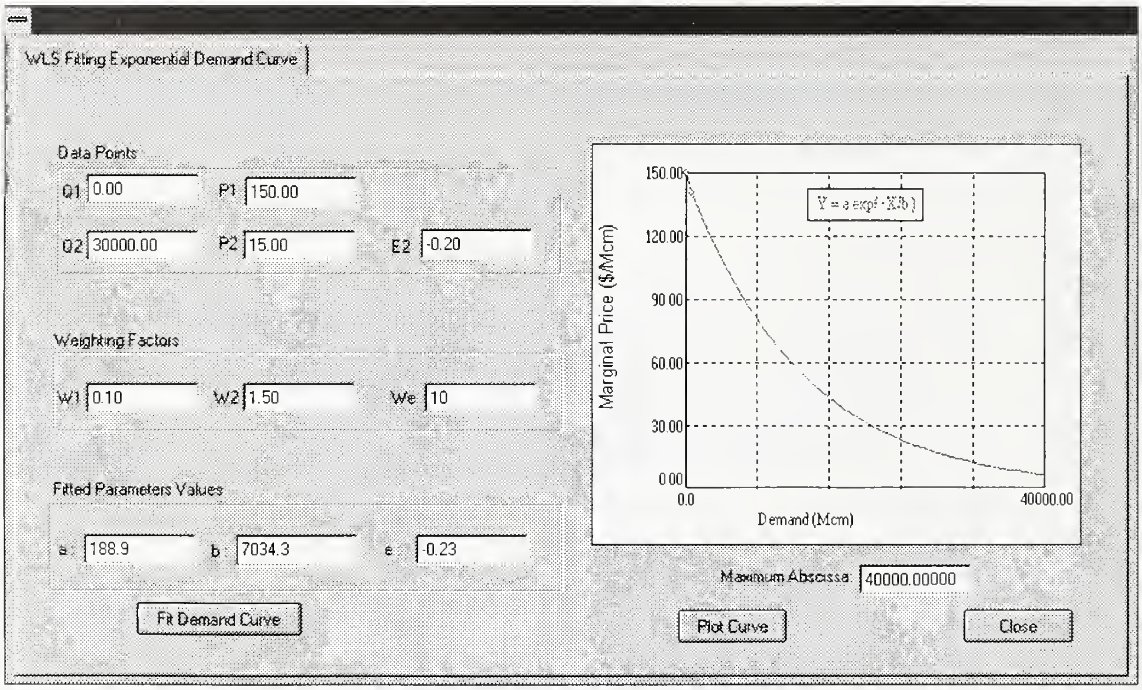

Figure B.2 Graphical user interface (GUI) for fitting an exponential demand curve.

The Data Points window includes the user specified price-quantity data for two points: $\{Q_1, P_1\} = \{0, 150\}$ and $\{Q_2, P_2\} = \{30000, 15\}$: and an estimated regional elasticity $\epsilon_2 = -0.20$. When the set of weights w_1, w_2, and w_3 are entered as $\{1, 1, 0\}$ (with zero weight for the regional elasticity), the solution is trivial (Case I) yielding $a = 150$, $b = 13029$, and the resulting elasticity at Q_2 is $\epsilon_2 = -0.434$. A second run assuming equal weights $\{1, 1, 1\}$ for the three pieces of information yields the parameters $a = 150.02$, $b = 12959.7$, and elasticity at Q_2 equal to -0.43.

Knowing the regional elasticity to be -0.2, the weighting coefficients are changed again to redirect the fitting algorithm to increase the compliance to the regional elasticity value. A new set of weights $\{0.1, 1.5, 10\}$ forces the LS fitting procedure to yield a demand function with parameters $a = 188.9$, $b = 7034.3$, and a computed elasticity $\epsilon_2 = -0.23$ (figure B.2). Note that this value approximates the regional elasticity at the expense of the quality of fitting of the two data points.

Appendix C

Explicit Modeling of Reservoir Evaporation

Chapter 5, Modeling Uncontrolled Releases, describes the approach used in AQUARIUS V96 to account for evaporation losses during the optimization process. This appendix presents an alternative formulation in which evaporation losses are not assumed known at the beginning of each optimization sequence but are included explicitly in the model formulation. In this manner, the model strives to minimize reservoir evaporation losses at the same time that water demands are meet. This different formulation brings about changes in the expressions for reservoir physical constraints and the benefit function associated with a variable-head powerplant. This new approach will be incorporated in the next version of the model.

Linearization of Reservoir Evaporation

According to Chapter 3, Storage Reservoirs, reservoir evaporation losses are computed by multiplying the surface area of the lake, using (3.4b), times the seasonal evaporation rate, denoted here by e_i [L/T]. The power function that relates reservoir area A [L^2] versus reservoir storage S [L^3] can be linerized as shown in figure C.1 such that evaporation losses E [L^3] during any given time interval i can be expressed as:

$$E_i = e_i\, A_i = e_i\, a + e_i\, b\, S_i \qquad \text{(C.1)}$$

where a and b are the coefficients of the linear model, computed automatically by the model using a weighted least-square method. For convenience, (C.1) is rewritten as:

$$E_i = w_i + v_i\, S_i \qquad \text{(C.2)}$$

where $w_i = e_i\, a$ and $v_i = e_i\, b$.

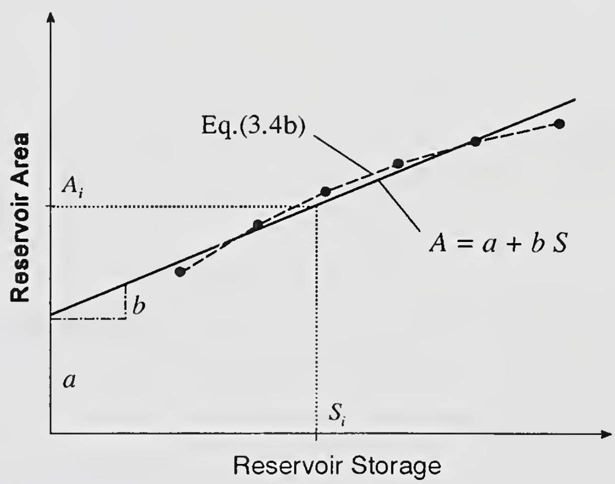

Figure C.1 Linearization of the reservoir area-storage relation.

Using the general notation introduced in Chapters 3 and 5, and adopting the expressions for inflows and outflows to a reservoir given by (3.2) and (3.3), respectively, the storage of a reservoir at the end of the first period of operation S_1 is expressed as:

$$S_1 = S^o + I_1 - O_1 - [w_1 + v_1 \frac{(S^o + S_1)}{2}] \tag{C.3}$$

where S^o is the storage of the reservoir at the beginning of the simulation, I_1 indicates all inflows to the reservoir (controlled and uncontrolled), O_1 indicates all outflows from the reservoir (controlled and uncontrolled) except for evaporation losses, which has been written separately within the term in brackets. Evaporation for the period is computed as a function of the average storage during the time interval. Rearranging terms in (C.3) yields:

$$S_1 = \frac{(1 - \frac{v_1}{2})}{(1 + \frac{v_1}{2})} S^o + \frac{1}{(1 + \frac{v_1}{2})} (I_1 - O_1 - w_1) \tag{C.4}$$

in which again variables are grouped to simplify the notation,

$$S_1 = \alpha_1 S^o + \beta_1 (I_1 - O_1 - w_1) \tag{C.5}$$

where $\alpha_i = \frac{(1 - \frac{v_i}{2})}{(1 + \frac{v_i}{2})}$ *and* $\beta_i = \frac{1}{(1 + \frac{v_i}{2})}$ (C.6a), (C.6b)

Similarly, the reservoir storage at the end of the second period can be written as:

$$S_2 = \alpha_2 S_1 + \beta_2 (I_2 - O_2 - w_2) \tag{C.7}$$

Substituting S_1 above by the expression in (C.5), and after rearranging terms we obtain,

$$S_2 = \alpha_2 \alpha_1 S^o + \alpha_2 \beta_1 (I_1 - O_1 - w_1) + \beta_2 (I_2 - O_2 - w_2) \tag{C.9}$$

In general, reservoir storage at the end of any time period i can be expressed as:

$$S_i = (\prod_{k=1}^{i} \alpha_k)\, S^o + (\prod_{k=2}^{i} \alpha_k)\, \beta_1 (I_1 - O_1 - w_1) + (\prod_{k=3}^{i} \alpha_k)\, \beta_2 (I_2 - O_2 - w_2) + \dots\dots$$
$$\dots + (\prod_{k=i}^{i} \alpha_k)\, \beta_{i-1} (I_{i-1} - O_{i-1} - w_{i-1}) + \beta_i (I_i - O_i - w_i) \qquad \text{(C.10)}$$

Equation (C.8) can be written in a more compact form in which the products $\prod_k \alpha_k$ and β terms are grouped under the new symbol γ as indicated in (C.11):

$$S_i = \gamma_o^i S^o + \gamma_1^i (I_1 - O_1 - w_1) + \gamma_2^i (I_2 - O_2 - w_2) + \dots$$
$$\dots + \gamma_{i-1}^i (I_{i-1} - O_{i-1} - w_{i-1}) + \gamma_i^i (I_i - O_i - w_i) \qquad \text{(C.11)}$$

Equation (C.11), which computes reservoir storage at the end of any time interval of simulation as a function of inflows, outflows and evaporation parameters, can also be expressed in matrix form as shown in (C.12):

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ \cdot \\ \cdot \\ S_i \end{bmatrix} = \begin{bmatrix} \gamma_0^1 \\ \gamma_0^2 \\ \gamma_0^3 \\ \cdot \\ \cdot \\ \gamma_0^i \end{bmatrix} S^o + \begin{bmatrix} \gamma_1^1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \gamma_1^2 & \gamma_2^2 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \gamma_1^3 & \gamma_2^3 & \gamma_3^3 & 0 & 0 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \gamma_1^i & \gamma_2^i & \gamma_3^i & \gamma_4^i & \gamma_5^i & \cdots & \gamma_{i-1}^i & \gamma_i^i \end{bmatrix} \begin{bmatrix} I_1 - O_1 - w_1 \\ I_2 - O_2 - w_2 \\ I_3 - O_3 - w_3 \\ \cdot \\ \cdot \\ I_i - O_i - w_i \end{bmatrix} \qquad \text{(C.12)}$$

Modified Reservoir Physical Constraints

Reservoir constraint equations similar to those presented in Chapter 5 can be derived for the case when evaporation losses are explicitly included in the formulation of the model as demonstrated in subsection C.1. The left-hand sides of the constraint equations listed below group all the decision variables (unknown terms), whereas the right-hand sides contain only the known terms. Note that the variable E is no longer part of the right-hand side terms.

Maximum reservoir storage. This inequality replaces the constraint in equation (5.10)

$$\sum_{k=1}^{i} \gamma_k^i \underline{d}_k^u + \sum_{k=1}^{i} \gamma_k^i \underline{d}_k^s \geq (\gamma_0^i S^o - S_M) + \sum_{k=1}^{i} \gamma_k^i (\underline{MR}_k^u + \underline{NF}_k^u + \underline{L}_k^u) - \sum_{k=1}^{i} \gamma_k^i (MR_k^s + L_k^s + w_k^s) \tag{C.13}$$

Minimum reservoir storage. This inequality replaces the constraint in equation (5.11)

$$\sum_{k=1}^{i} \gamma_k^i \underline{d}_k^u + \sum_{k=1}^{i} \gamma_k^i \underline{d}_k^s \geq (S_m - \gamma_0^i S^o) - \sum_{k=1}^{i} \gamma_k^i (\underline{MR}_k^u + \underline{NF}_k^u + \underline{L}_k^u) + \sum_{k=1}^{i} \gamma_k^i (MR_k^s + L_k^s + w_k^s) \tag{C.14}$$

Reservoir final storage. This equality replaces the constraint in equation (5.12)

$$\sum_{k=1}^{np} \gamma_k^i \underline{d}_k^u + \sum_{k=1}^{np} \gamma_k^i \underline{d}_k^s = (S^f - \gamma_0^{np} S^o) - \sum_{k=1}^{np} \gamma_k^i (\underline{MR}_k^u + \underline{NF}_k^u + \underline{L}_k^u) + \sum_{k=1}^{np} \gamma_k^i (MR_k^s + L_k^s + w_k^s) \tag{C.15}$$

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